## Cosmic Microwave Background !

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## What is the CMB

$$
1+z=\frac{\lambda_{\text {now }}}{\lambda_{\text {em }}}=\frac{r_{\text {now }}}{r_{e m}}
$$



$$
\begin{aligned}
& \|_{\mathrm{T}}^{10^{-6} \mathrm{~S}} \mathrm{GeV} \quad b+\bar{b} \rightarrow 2 \gamma . \\
& \begin{array}{l}
10^{13} \mathrm{~S} \\
\mathrm{~T}=3000 \mathrm{~K}
\end{array}
\end{aligned}
$$

According to modern cosmology:
An abundant background of photons filling the Universe. - Generated in the very early universe, less than $4 \mu$ s after the Big Bang ( $10^{9} \gamma$ for each baryon) from a small $b-\bar{b}$ asymmetry

- Thermalized in the primeval fireball (in the first 380000 years after the big bang) by repeated scattering against free electrons
- Redshifted to microwave frequencies ( $\mathrm{z}_{\mathrm{CMB}}=1100$ ) and diluted in the subsequent 14 Gyrs of expansion of the Universe



Environment! $\boldsymbol{<} \begin{aligned} & \text { Space : to remove atmospheric emission } \\ & \text { Cryogenics : to limit instrumental emission }\end{aligned}$

## COBE-FIRAS

- COBE-FIRAS was a cryogenic MartinPuplett FourierTransform
Spectrometer with composite bolometers. It was placed in a 400 km orbit.
- A null instrument comparing the specific ${ }^{\circ}$ sky brightness to the brightness of a cryogenic Blackbody



All this is cooled at $2 \mathrm{~K}\left(-271^{\circ} \mathrm{C}\right)$

## Fourier Transform Spectrometers (FTS)

- To measure spectra, you use interference (prism, grating, FP ...)
- In the case of the FTS only 2 light beams interfere: this is the simplest experimental configuration, but results in a complex encoding of the spectrum.


## Recipe for a FTS

- Take the beam to be analyzed (A), transform it into a quasi-parallel beam (C), and split it in two beams (D).
- Delay one of the two beams (E), driving it along a longer optical path (x).
- Recombine the beams on the detector (H and J), and record the detected power vs. the optical path difference (this is called the interferogram).
- The interferogram is the Fourier transform of the spectrum of the incoming radiation (as shown below).



## Elementary theory of the FTS

- The OPD (optical path difference) is $2 x$.
- For a perfectly monochromatic radiation with wavenumber $\sigma(=1 / \lambda)$ the resulting field on the detector will be

$$
\begin{aligned}
E(t) & =E_{o}(\sigma) R T(\sigma) \cos (2 \pi \sigma \sigma t)+ \\
& +E_{o}(\sigma) R T(\sigma) \cos (2 \pi \sigma c t+4 \pi \sigma x)
\end{aligned}
$$

- Here RT is the efficiency of the beamsplitter (frequency dependent, in general)



## Elementary theory of the FTS $E(t)=E_{o}(\sigma) R T(\sigma) \cos (2 \pi \sigma c t)+E_{o}(\sigma) R T(\sigma) \cos (2 \pi \sigma c t+4 \pi \sigma x)$

- The power on the detector $I(x)$ will be proportional to the mean square electrical field:
$I(x) \propto\left\langle E(t)^{2}\right\rangle=\left\langle E(t) E^{*}(t)\right\rangle=$
$=E_{o}^{2}(\sigma) r t(\sigma)\left[e^{i 2 \pi \sigma c t}+e^{i 2 \pi \sigma t t} e^{i 4 \pi \sigma x}\left[e^{-i 2 \pi \sigma t t}+e^{-i 2 \pi \sigma t} e^{-i 4 \pi \sigma x}\right]=\right.$
$=E_{o}^{2}(\sigma) r t(\sigma)\left[1+e^{-i 4 \pi \sigma x}+e^{i 4 \pi \sigma x}+1\right]=E_{o}^{2}(\sigma) r t(\sigma) 2(1+\cos 4 \pi \sigma x)$
- So the interferogram is $I(x)-\langle I\rangle=r t(\sigma) \cos (4 \pi \sigma x)$
- If the input radiation is not monochromatic, and each wavenumber has amplitude $S(\sigma)$ :

$$
I(x)-\langle I\rangle=\int_{0}^{\infty} S(\sigma) r t(\sigma) \cos (4 \pi \sigma x) d \sigma
$$

The specific Brightness and the interferogram are related by a Fourier Transform


## Elementary theory of the FTS

$$
S(\sigma) r t(\sigma)=\int_{-\infty}^{\infty}(I(x)-\langle I\rangle) \cos (4 \pi \sigma x) d x
$$

- For obvious reasons we cannot extend $x$ to infinity! If the maximum displacement of the moving mirror is $x_{\max }$, all we can do is to compute

$$
S^{\prime}(\sigma) r t(\sigma)=\int_{-x}^{x_{\max }}(I(x)-\langle I\rangle) \cos (4 \pi \sigma x) d x
$$

- $\mathrm{S}^{\prime}$ is an approximation of the real spectrum S
- The main difference is in the effective spectral resolution of the spectrometer, which for $S^{\prime}$ is limited to approx. $1 /\left(2 x_{\max }\right)$.


## Spectral Resolution

- Consider a monochromatic line with wavenumebr $\sigma_{0}$ : the interferogram is

$$
I(x)-\langle I\rangle=I_{o} \cos \left(4 \pi \sigma_{o} x\right)
$$

- The Fourier integral, limited to $\pm \mathrm{x}_{\max }$, is:

$$
\begin{aligned}
& S^{\prime}(\sigma)=I_{o} \int_{-x_{\max }}^{x_{\max }} \cos \left(4 \pi \sigma_{o} x\right) \cos (4 \pi \sigma x) d x \quad \Rightarrow \\
& S^{\prime}(\sigma)=I_{o} x_{\max } \frac{\sin 4 \pi\left(\sigma-\sigma_{o}\right) x_{\max }}{4 \pi\left(\sigma-\sigma_{o}\right) x_{\max }}
\end{aligned}
$$

- This is an approximation of the real $S(\sigma)$ which would be a delta function centered in $\sigma_{0}$ : in place of a delta, we get a sinc, with a half-width approx. $1.23 /\left(2 x_{\max }\right)$.



## Beamsplitter problems

$$
S^{\prime}(\sigma) r t(\sigma)=\int_{-x_{\max }}^{x_{\max }}(I(x)-\langle I\rangle) \cos (4 \pi \sigma x) d x
$$

- What we get is the input spectrum times the efficiency of the beamsplitter.
- If the latter goes to zero, we cannot retrieve the spectrum.
- So we need good beamplitters, ideally with $r t=0.25$, independent on frequency.
- The simplest


## the beamsplitter

 beamsplitter is a dielectric slab, with refaction index n and thickness t.- Due to multiple reflections inside the slab, the transmitted and reflected fields can be computed as the sum of an infinite number of components with decreasing amplitude (a converging series) and increasing phase delay.
$\delta=4 \pi n d \cos \theta^{\prime} \sigma$

$E=E_{o}\left(-r \cos 2 \pi \sigma c t+r t^{2} \cos (2 \pi \sigma c t+\delta)+r^{3} t^{2} \cos (2 \pi \sigma c t+2 \delta)+\right.$
$+r^{5} t^{2} \cos (2 \pi \sigma c t+3 \delta)+\ldots$
From this, the efficiency $\mathrm{rt}(\sigma)$ is computed


## the beamsplitter

- The efficiency is a periodic function with zeros at wavenumbers m
$\sigma_{m}=\overline{n d \cos \theta^{\prime}}$
- Whatever thickness and refraction index you select, this is not efficient at low frequencies.



## Linear Polarizers

- Linear polarizers can be used as high efficiency achromatic beamsplitters at long wavelengths.
- A linear polarizer is an optical device transmitting only the projection of the E field of the EM wave parallel to a given direction, which is called the principal axis of the polarizer.
- Unpolarized radiation (where the E field direction in the wavefront is random) is transformed into linearly polarized radiation (where the E field direction is constant) when crossing a polarizer.



## Linear Polarizers

- The transmitted field E' can be computed projecting the incoming field E along the Principal Axis:
$E_{p a r}=E_{x} \cos \phi+E_{y} \sin \phi$
$\left\{E_{x}^{\prime}=E_{p a r, x}=\left(E_{x} \cos \phi+E_{y} \sin \phi\right) \cos \phi \quad \mathrm{x}\right.$
$E_{y}^{\prime}=E_{p a r, y}=\left(E_{x} \cos \phi+E_{y} \sin \phi\right) \sin \phi$
- The component of the incoming field orthogonal to the PA is either absorbed or reflected, depending on the perticular polarizer used.
- Note that

$$
I^{\prime}=|E|^{2}=\left|E_{p a r}\right|^{2}=|E|^{2} \cos ^{2} \theta=I \cos ^{2} \theta \quad \text { (Malus law) }
$$

- So, for $\theta=45^{\circ}$ incidence, half of the intensity is transmitted (and half is reflected or absorbed).
- At long wavelengths metallic wire grids act as ideal polarizers:
- Radiation with E parallel to the wires induces a current in the wires, so the polarizer acts as a metallic mirror: radiation is fully reflected and is not transmitted.
- Radiation with E orthogonal to the wires cannot induce a current in the wires, so it is transmitted.
- Radiation at a generic angle from the wires is partially transmitted (orthogonal component) and partially reflected (parallel component).
- If the spacing of the wires $a$ and their diameter $d$ are much less than the wavelength, the wire grid is very close to an ideal polarizer, with its principal axis orthogonal to the wires.
- Wire grid polarizers can be used as ideal beamsplitters for radiation at $45^{\circ}$ wrt the wires: half of the intensity is transmitted, and half is reflected, without any wavelength dependance.
- Wire grids can be machined easily using a lathe and tungsten wire, which is available in long coils.


- In the Martin-Puplett configuration, radiation is prepared by the first polarizer, then is split by the second, and is recombined by the third.
- There are two sources. The beam from source $1^{\prime}$ is reflected one time more than the beam from source 1.
- For each metallic reflection there is a $180^{\circ}$ phase change of the electric field, so the detector will measure the $S_{1}$ difference in spectral brightness between source 1 and source 1'.
- The instrument becomes a zero instrument, comparing the brightness of two sources (see later).
- In the case of FIRAS one source was the sky, the other one was an internal blackbody.


## Martin Puplett Interferometer

- Two input ports and two output ports.
- Uses two unpolarized sources, and two detectors sensitive to the power.
- Let's study the operation following the
 beams.


## Martin Puplett Interferometer

- The input polarizer A reflects radiation from source $\mathbf{S}_{\mathbf{1}}$, and transmits radiation from $\mathbf{S}_{1}$
- Assume that the polarizer wires are horizontal (main axis of the polarizer vertical)
- The beam at position 3 has a vertical component from $\mathbf{S}_{\mathbf{1}}$ and a
 horizontal
component from $\mathbf{S}_{\mathbf{1}}$,


## Martin Puplett Interferometer

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 horizontal
component from $\mathbf{S}_{\mathbf{1}}$,


## Martin Puplett Interferometer

- The wires of the beamsplitter polarizer B are oriented to be seen by incoming radiation at $45^{\circ}$ from the drawing plane.
- In this way, B reflects a fraction of the vertical component from $\mathbf{S}_{1}$ and a fraction of the horizontal component from $\mathbf{S}_{\mathbf{1}^{\prime}}$.
- So beam 4 will be polarized at $45^{\circ}$ and will consist of equal contributions from
 both $\mathbf{S}_{\mathbf{1}}$ e da $\mathbf{S}_{\mathbf{1}}$.



## Martin Puplett Interferometer

- In addition, B transmits a fraction of the vertical component from $\mathbf{S}_{\mathbf{1}}$ and a fraction of the horizontal polarization from $\mathbf{S}_{\mathbf{1}} \cdot \mathbf{S}_{\mathbf{1}}$
- So beam 4' is polarized at $45^{\circ}$ and consists of equal contributions from both $\mathbf{S}_{\mathbf{1}}$ and $\mathbf{S}_{\mathbf{1}}$,




## Martin Puplett Interferometer

- The roof mirror CD reflects beam 4 into beam 6 back to the beamsplitter B.
- A roof mirrror is used in place of a normal mirror because it rotates the polarization plane by $90^{\circ}$.
- In this way beam 6, which had been reflected by B, now is transmitted towards the
 detectors.



## Martin Puplett Interferometer

- In the same way the roof mirror C'D' reflects beam 4' back towards the beamsplitter (as 6')
- The polarization plane is rotated by $90^{\circ}$, so that beam 6', which had been transmitted (as 4') now is
 reflected towards the detectors.


## Martin Puplett Interferometer

- The output polarizer G has wires parallel to the drwaing plane.
- The rays coming from the beamsplitter, which are at $45^{\circ}$, have both horizontal and vertical components (coming from both sources) so they contribute to both the beams towards the detectors (both transmitted and reflected)
- In this way both detectors receive radiation from both sources, which passed through both the arms of the interferometer.



## Martin Puplett Interferometer

- The fundamental difference is that radiation from source $\mathbf{S}_{\mathbf{1}}$, underwent 4 reflections, while radiation from $\mathbf{S}_{\mathbf{1}}$ underwent only 3 reflections. Since each reflection produces a $180^{\circ}$ phase shift, the instrument measures the diffence between
interferograms produced by $\mathbf{S}_{\mathbf{1}}$, and $\mathbf{S}_{\mathbf{1}}$


## Quantitative treatment: uses Jones Calculus

- Jonex matrices are used to describe linearly polarized radiation (Jones 1941)
- The interaction of the E field of the EM wave with an optical component is described by a $2 x 2$ matrix:

$$
\binom{E_{x, \text { OUT }}}{E_{y, \text { OUT }}}=\left(\begin{array}{ll}
J_{11} & J_{12} \\
J_{21} & J_{22}
\end{array}\right)\binom{E_{x, I N}}{E_{y, I N}}
$$

- They work only for fully polarized radiation. For partially polarized radiation one can use Muller calculus (loosing any phase information).


## E field If $E$ is the amplitude of the electric field of the EMW, it is represented as in the following examples:

- Linear polarization aligned along x axis

$$
\begin{gathered}
E\binom{1}{0} \\
\left.\frac{E\binom{0}{1}}{\frac{E}{\sqrt{2}}\binom{1}{1}} \begin{array}{l}
\frac{E}{\sqrt{2}}\binom{1}{-i} \\
\frac{E}{\sqrt{2}}\binom{1}{-1} \\
i
\end{array}\right)
\end{gathered}
$$

- Linear polarization aligned along y axis
- $45^{\circ}$ from x axis
- $-45^{\circ}$ from x axis
- Circular polarization (right)
- Circular polarization (left)


## Reference system

- Comoving with the light beam :


## Mirrors



- Ideal single mirror, orthogonal to $x z$ plane:

$$
M=\left(\begin{array}{cc}
1 & 0 \\
0 & -1
\end{array}\right)
$$

- Ideal roof mirror, orthogonal to xz plane:

$$
R M=\left(\begin{array}{cc}
1 & 0 \\
0 & -1
\end{array}\right)\left(\begin{array}{cc}
1 & 0 \\
0 & -1
\end{array}\right)=\left(\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right)
$$

## Linear polarizers

- Transmission : Polarizer with horizontal principal axis:
- Transmission : Polarizer with vertical principal axis:

- Transmission :

Polarizer with principal axis at angle $\phi$ from x axis

$$
\begin{aligned}
& P_{t}(\varphi)=\left(\begin{array}{cc}
\cos ^{2} \varphi & \cos \varphi \sin \varphi \\
\cos \varphi \sin \varphi & \sin ^{2} \varphi
\end{array}\right) \\
& P_{r}(\varphi)=\left(\begin{array}{cc}
\sin ^{2} \varphi & -\cos \varphi \sin \varphi \\
\cos \varphi \sin \varphi & -\cos ^{2} \varphi
\end{array}\right)
\end{aligned}
$$

- Reflection : Polarizer with principal axis at angle $\phi$ from x axis


## Delay

- Introduced by an optical path difference $\delta=4 \pi \sigma x$ : this is common for both polarizations, so

$$
D(\delta)=\left(\begin{array}{cc}
e^{i \delta} & 0 \\
0 & e^{i \delta}
\end{array}\right)
$$

- The two sources $S_{1}$ and $S_{1}$, are described by Jones vectors

$$
S_{1}=\binom{A_{x}}{A_{y}}
$$

$$
S_{1}^{\prime}=\binom{B_{x}}{B_{y}}
$$

- beam 3 , after the

input polarizer
(with horizontal principal axis), is

$$
S_{3}=P_{t}(0) S_{1}+P_{r}(0) S_{1}^{\prime}=\binom{A_{x}}{-B_{y}}
$$

- beam 4 (reflected by the
beamsplitter) and beam 4'
(transmitted by the beamsplitter) will be:


$$
\begin{aligned}
& S_{4}=P_{r}(\pi / 4) S_{3}=\frac{1}{2}\left(\begin{array}{ll}
1 & -1 \\
1 & -1
\end{array}\right)\binom{A_{x}}{-B_{y}}=\frac{1}{2}\binom{A_{x}+B_{y}}{A_{x}+B_{y}} \\
& S_{4}^{\prime}=P_{t}(\pi / 4) S_{3}=\frac{1}{2}\left(\begin{array}{ll}
1 & 1 \\
1 & 1
\end{array}\right)\binom{A_{x}}{-B_{y}}=\frac{1}{2}\binom{A_{x}-B_{y}}{A_{x}-B_{y}}
\end{aligned}
$$

- Since roof mirrors are represented by unity matrix, we have also

$$
S_{6}=S_{4}=\frac{1}{2}\binom{A_{x}+B_{y}}{A_{x}+B_{y}}
$$


$S_{6}^{\prime}=D S_{4}^{\prime}=D \frac{1}{2}\binom{A_{x}-B_{y}}{A_{x}-B_{y}}=\frac{1}{2}\binom{\left(A_{x}-B_{y}\right) e^{i \delta}}{\left(A_{x}-B_{y}\right) e^{i \delta}}$

- $\mathrm{S}_{6}$ is transmitted by the beamsplitter, while $\mathrm{S}_{6}$ ' is reflected, so

$$
S_{8}=P_{t}(\pi / 4) S_{6}+P_{r}(3 \pi / 4) S_{6}^{\prime}=\frac{1}{2}\left(\begin{array}{ll}
1 & 1 \\
1 & 1
\end{array}\right) S_{6}+\frac{1}{2}\left(\begin{array}{cc}
1 & 1 \\
-1 & -1
\end{array}\right) S_{6}^{\prime}
$$

$$
\begin{aligned}
& S_{8}=P_{t}(\pi / 4) S_{6}+P_{r}(3 \pi / 4) S_{6}^{\prime}=\frac{1}{2}\left(\begin{array}{ll}
1 & 1 \\
1 & 1
\end{array}\right) S_{6}+\frac{1}{2}\left(\begin{array}{cc}
1 & 1 \\
-1 & -1
\end{array}\right) S_{6}^{\prime} \\
& \text { - So } \\
& S_{8}=\frac{1}{2}\binom{A_{x}\left(e^{i \delta}+1\right)-B_{y}\left(e^{i \delta}-1\right)}{-A_{x}\left(e^{i \delta}-1\right)+B_{y}\left(e^{i \delta}+1\right)} \\
& \text { - After the interaction with the } \\
& \text { output beamsplitter the beams to } \\
& \text { the detectors are } \\
& S_{9}=P_{t}(0) S_{8}=\frac{1}{2}\left(A_{x}\left(e^{i \delta}+1\right)-B_{y}\left(e^{i \delta}-1\right)\right. \\
& S_{9}^{\prime}=P_{r}(0) S_{8}=\frac{1}{2}\binom{D_{x}}{A_{x}\left(e^{i \delta}-1\right)-B_{y}\left(e^{i \delta}+1\right)}
\end{aligned}
$$

- Now we can compute the power on the detectors:

$$
\begin{aligned}
& I=E_{x} E_{x}^{*}+E_{y} E_{y}^{*} \rightarrow \\
& I_{9}=\frac{1}{2}\left[A_{x}^{2}(1+\cos \delta)+B_{y}^{2}(1-\cos \delta)\right]=\frac{A_{x}^{2}+B_{y}^{2}}{2}+\frac{A_{x}^{2}-B_{y}^{2}}{2} \cos \delta \\
& I_{9}^{\prime}=\frac{1}{2}\left[A_{x}^{2}(1-\cos \delta)+B_{y}^{2}(1+\cos \delta)\right]=\frac{A_{x}^{2}+B_{y}^{2}}{2}-\frac{A_{x}^{2}-B_{y}^{2}}{2} \cos \delta
\end{aligned}
$$

- Both detectors measure a constant intensity (equal to half of the sum of the intensities from the two sources), plus a modulated intensity (modulated by the optical path difference), whose amplitude is the difference of the spectra from the two sources.
- The interferogram is zero if the two sources have the same spectrum.


$$
\begin{aligned}
& I_{S K Y}(x)=C \int_{0}^{\infty}\left[S_{S K Y}(\sigma)-S_{R E F}(\sigma)\right] r t(\sigma)\{1+\cos [4 \pi \sigma x]\} d \sigma \\
& I_{C A L}(x)=C \int_{0}^{\infty}\left[S_{C A L}(\sigma)-S_{R E F}(\sigma)\right] r t(\sigma)\{1+\cos [4 \pi \sigma x]\} d \sigma
\end{aligned}
$$

- To measure a few K blackbody, you need a cryogenic reference blackbody, with variable temperature. Otherwise you do not null the signal.
- Practical design of a Blackbody cavity: see e.g. Quinn T.J., Martin J.E., (1985), Phil. Trans. R. Soc. Lond., A316, 85: who made a radiometric measurement of the Boltzmann constant (precise to 5 significant digits!)



## FIRAS

- The FIRAS guys were able to change the temperature of the internal blackbody until the interferograms were flat zero.
- This is a null measurement, which is much more sensitive than an absolute one: (one can boost the gain without saturating !).
- This means that the difference between the spectrum of the sky and the spectrum of a blackbody is zero, i.e. the spectrum of the sky is a blackbody with the same temperature as the internal reference blackbody.
- This also means that the internal blackbody is a real blackbody: it is unlikely that the sky can have the same deviation from the Planck law as the source built in the lab.

- Isotropic expansion or contraction: For every observer : $r=$ physical distance $\chi=$ comoving distance $a(t)=$ scale factor
- FRW metric: the most general homogenous isotropic metric


$$
(d s)^{2}=c^{2} d t^{2}-a^{2}(t)\left[\left(\frac{d \chi}{\sqrt{1-k \chi^{2}}}\right)^{2}-(\chi d \theta)^{2}-(\chi \sin \theta d \varphi)^{2}\right]
$$

- $1 / k^{2}=$ curvature of space
- If we want to know how the universe expands, we need to integrate the Einstein Field Equations

| Curvature Tensor <br> (derived from the <br> metric of the universe) |
| :--- |

$$
G=-\frac{8 \pi G}{c^{4}} T
$$

Stress-energy tensor (describes the energy content of the universe)

## Evolution of the scale factor

- $a(t)$ is the solution of the Friedmann equation, which is found from Einstein's field equations for a homogeneous isotropic medium, and can be rewritten:

$$
\left(\frac{\dot{a}}{a}\right)^{2}=H_{o}^{2}\left\{\Omega_{R o} a^{-4}+\Omega_{M o} a^{-3}+\Omega_{K o} a^{-2}+\Omega_{\Lambda}\right\}
$$

- The solution $a(t)$ depends on the different kinds of energy densities relevant at the considered epoch. To integrate, we need the $\Omega \mathrm{s}$ :

$$
\Omega_{o i}=\frac{\rho_{o i}}{\rho_{c, o}} ; \quad \rho_{c, o}=\frac{3 H_{o}^{2}}{8 \pi G}
$$

- The very first result is that the universe cannot be static: $a=a(t)$.
- From observations we know that the universe expands today, so $a(t)$ is growing today.
- To say more on the previous and future behaviour of the scale factor we need to estimate the cosmological parameters $\Omega_{i}$ and $H_{o}$.
- Cosmology recently entered in a "precision" phase, were the cosmological parameters have been estimated with good precision.
- Precision measurements of the CMB played a key role in this process.


## Radiation Phase

- From Friedmann equation is evident that at early epochs ( $a$ small) the expansion is driven by radiation:

$$
\left(\frac{\dot{a}}{a}\right)^{2}=H_{o}^{2}\left\{\Omega_{R o} a^{-4}+\Omega_{M o} a^{-3}+\Omega_{K o} a^{-2}+\Omega_{\Lambda}\right\} \approx H_{o}^{2} \Omega_{R o} a^{-4}
$$

- Note that the expansion rate $\dot{a} / a$ tends to infinity at the beginning (near the Big Bang), and then decreases with time.
- In this phase the solution $a(t)$ can be found analytically:

$$
\frac{a^{2}}{2}=H_{o} \sqrt{\Omega_{R o}} t \Rightarrow a(t)=\left\{2 \sqrt{\Omega_{R o}}\right\}^{1 / 2}\left(H_{o} t\right)^{1 / 2}
$$

- We know $\mathrm{H}_{0}$ from Hubble's law. $\Omega_{\mathrm{R}}$ has contributions from CMB photons, but also from all other relativistic particles present at early epochs. So the extrapolation using only the energy density of the CMB would not be precise.


Note: at the end of the radiation phase the temperature $\mathrm{T}=\mathrm{T}_{\mathrm{CMB}} / a$ was still $>10^{5} \mathrm{~K}$ The universe was still ionized and opaque.

## Matter Phase

- When the energy of non.relativistic matter becomes dominant

$$
\left(\frac{\dot{a}}{a}\right)^{2}=H_{o}^{2}\left\{\Omega_{R o} a^{-4}+\Omega_{M o} a^{-3}+\Omega_{K o} a^{-2}+\Omega_{\Lambda}\right\} \quad \approx H_{o}^{2} \Omega_{M o} a^{-4}
$$

- In this phase the solution $a(t)$ can be found analytically:
$\left.\frac{2}{3} a^{3 / 2}\right|_{a_{o}} ^{a}=H_{o} \sqrt{\Omega_{m o}}\left(t-t_{o}\right) \Rightarrow$

$a(t)=\left(\frac{3}{2} \sqrt{\Omega_{\mathrm{Mo}}} H_{o}\left(t-t_{o}\right)+a_{o}^{3 / 2}\right)^{2 / 3}$

From this equation we can estimate how long it took to go from $a=10^{-5}$ (end of radiation phase) to $a=10^{-3}$ (recombination).
The result is 380000 years.
This number is important for the following.


## Expansion vs Horizon



## Horizons at recombination



## Horizons at recombination



## Horizon

- The physical phenomena happening within the causal horizon ar different from the phenomena at scales outside the horizon.
- Forces are transmitted at most at the speed of light, so phenomena outside the causal horizon are frozen until they enter the horizon.
- We should be able to see the effects of causal horizons, impressed in the image of the CMB.


## Horizons

- At recombination ( $\mathrm{t}=380000$ years), only regions of the Universe closer than 380000 light years have had the possibility (enough time) to interact.
- That length, as seen from a distance of the order of $\mathrm{c} / \mathrm{H}_{\mathrm{o}}=14$ Glyrs, has an angular size of about 1 degree.
- They might be very different, since they could not interact during all the previous history of the Universe, from the Big Bang to recombination !
- However, measurements show that this is not the case.
- CMB anisotropy measurements.
- Anisotropy means that brightness is a function of the observed direction.
- Large brightness variations


## Measuring anisotropy

 would be evident in a map.- However, since its discovery, it was evident that the CMB is a very isotropic background.
- At a few mm of wavelength, the brightness of the sky is dominated by the CMB, and is isotropic to at least 1 part over 100.
- Only if we remove the average brightness and increase the contrast of the image, we start to see structures.
- But this requires special experimental configurations,
 removing a large background an enhancing small fluctuations.


## Measuring anisotropy

- A way to remove the common-mode signal is to alternate on the detector two contiguous regions of the sky, and measure the AC signal removing the DC signal electronically.
- A small AC signal synchronous to the modulation can be extracted from detector noise using a
 demodulation technique (a lock-in amplifier).


## Lock-in

- Using a modulation/demodulation technique has 3 purposes:
- Remove the common mode signal (instrumental and atmospheric emission - to first order)
- Remove noise at frequencies different from the modulation frequency
- Produce a signal proportional to the Difference of Brightness between the two observed regions.
$P_{A}=A \Omega E R\left(B_{\text {instr }}+\left(1-t_{\text {atm }}\right) B_{\text {atmosph }}+t_{\text {atm }} B_{s}(A)\right)$
$P_{B}=A \Omega E R\left(B_{\text {instr }}+\left(1-t_{\text {atm }}\right) B_{\text {atmosph }}+t_{\text {atm }} B_{s}(B)\right)$
$\Delta P=A \Omega E R t_{\text {atm }}\left\{B_{s}(A)-B_{s}(B)\right\}$


## ${ }^{\uparrow}$ Signal from detector <br> Lock-in



$$
S_{\text {out }}=\langle S(t) R(t)\rangle=\frac{1}{2}\left\langle S_{A} \times 1\right\rangle+\frac{1}{2}\left\langle S_{B} \times-1\right\rangle=\frac{1}{2}\left[S_{A}-S_{B}\right]
$$

## Lock-in



In presence of noise :

$$
\begin{aligned}
& S_{\text {out }}=\langle[S(t)+n(t)] R(t)\rangle= \\
& =\frac{1}{2}\left\langle S_{A} \times 1\right\rangle+\frac{1}{2}\left\langle S_{B} \times-1\right\rangle+\frac{1}{2}\langle n(t) \times 1\rangle+\frac{1}{2}\langle n(t) \times-1\rangle= \\
& =\frac{1}{2}\left[S_{A}-S_{B}\right]+n^{\prime}
\end{aligned}
$$

$n$ ' is a zero-average noise: due to the lack of correlation between $n(t)$ and $R(t), n$ ' tends to 0 if the average period is long enough (many modulation cycles).
To quantify, we need to specify the noise better.
The combination multiplier + integrator is equivalent to a band-pass filter centered on the reference frequency.

- Let's consider for simplicity a sine-wave signal :

$$
S(t)=V_{s} \cos \left(\omega_{s} t+\varphi_{s}\right)
$$

- The signal from the sky will be at the same frequency of the reference, while the noise will have contributions at all frequencies.
- The reference also can be a sine-wave at the same frequency as the signal:

$$
R(t)=\cos \left(\omega_{R} t+\varphi_{R}\right)
$$

- At the output of the multiplier we get

$$
V_{p}(t)=V_{s} \cos \left[\left(\omega_{s}+\omega_{R}\right) t+\left(\varphi_{s}+\varphi_{R}\right)\right]+V_{s} \cos \left[\left(\omega_{s}-\omega_{R}\right) t+\left(\varphi_{s}-\varphi_{R}\right)\right]
$$

- If the low-pass filter has a cut-off frequency lower than $\mathrm{f}_{\mathrm{c}}=1 / \mathrm{T}<2 \pi / \omega$, the sum frequency will be cut, while the difference frequency will pass with an amplitude
$V_{s}|H| \omega_{s}-\omega_{R} \| \cos \left(\varphi_{s}-\varphi_{R}\right)$
- Where H is the tranfer function of the low-pass filter.

The sky signal is exactly at the same frequency as the reference, with no

$$
V_{\text {out }}=V_{S}\left|H\left(\left|\omega_{S}-\omega_{R}\right|\right)\right|
$$ phase delay, so it is transferred to the output as it is.

Noise contributes to all frequencies, but only those within the bandpass


If $w_{V}$ is the spectral density of the noise (in $\mathrm{V}^{2} / \mathrm{Hz}$ ) :

$$
\begin{aligned}
& N_{\text {in }}=\sqrt{w_{V} f_{\max }} ; \quad N_{\text {out }}=\sqrt{\frac{w_{V}}{2 T}} \\
& \frac{S / N_{\text {out }}}{S / N_{\text {in }}}=\sqrt{2 T f_{\max }}
\end{aligned}
$$

## Early measurements of CMB anisotropy

- Using the beam-switching technique described above, you can integrate for some time on one couple of directions, then change directions and integrate again, then change $\ldots$ until you get a statistical sample of the sky, consisting typically in a few tens to a few hundreds sky temperature differences $\Delta T_{i}$.
- This is enough to estimate the rms fluctuation of the temperature (brightness) of the sky, extracting it from the noise.
- These experiments started immediately after the discovery of the CMB (Penzias and Wilson stated in the discovery paper of 1965 that it was isotropic to $10 \%$ )... and were very frustrating !
- For more than two decades a few groups of pioneers of the CMB improved their isotropometers, obtaining increasingly stringent upper limits for the anisotropy of the CMB, down to a level

$$
\frac{\Delta T}{T}<10^{-4}
$$

at scales of the order of the horizon. So regions which have never been in causal contact before produce the same CMB brightness, with outstanding precision. How is this possible? This is the paradox of horizons.

