

Course II:

Compact Objects, the Dynamic Radio Sky, and 21st Century Radio Telescope Facilities

Jim Cordes
Professor of Astronomy
Cornell University

Five Lectures:

1. Astrophysics of neutron stars and forefront science
2. Plasma propagation effects (ISM, IGM, ionosphere) and surveys
3. Precision astrometry for kinematics of compact objects
4. The dynamic radio sky (transient sources and variability)
5. New radio telescope arrays for key science and discovery



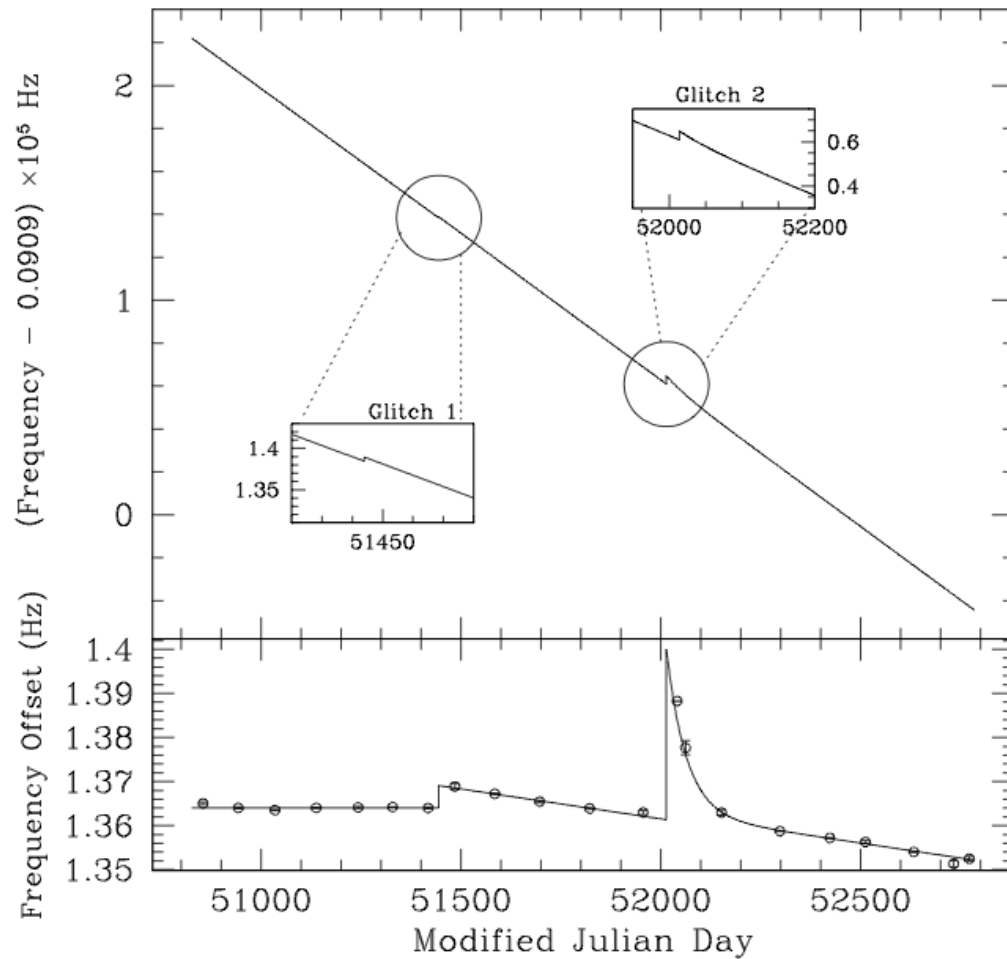
Cornell University

Astrophysics of neutron stars and other compact objects

- What are pulsars?
 - How were they discovered?
 - What are their basic properties?
 - How were they formed?
 - How do they emit coherent radio emission?
- What populations of neutron stars exist and how are they distributed in the Milky Way (and why)?
- How are pulsar surveys done? What is detectable?
- What are the scientific frontiers?
- How will current and future telescopes address the big questions?

Neutron Star Astrophysics

A NEUTRON STAR: SURFACE and INTERIOR



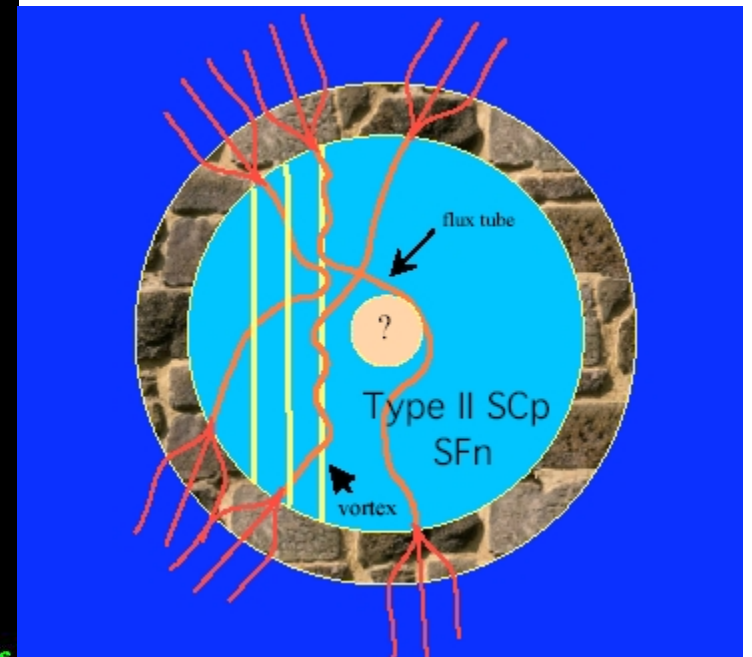
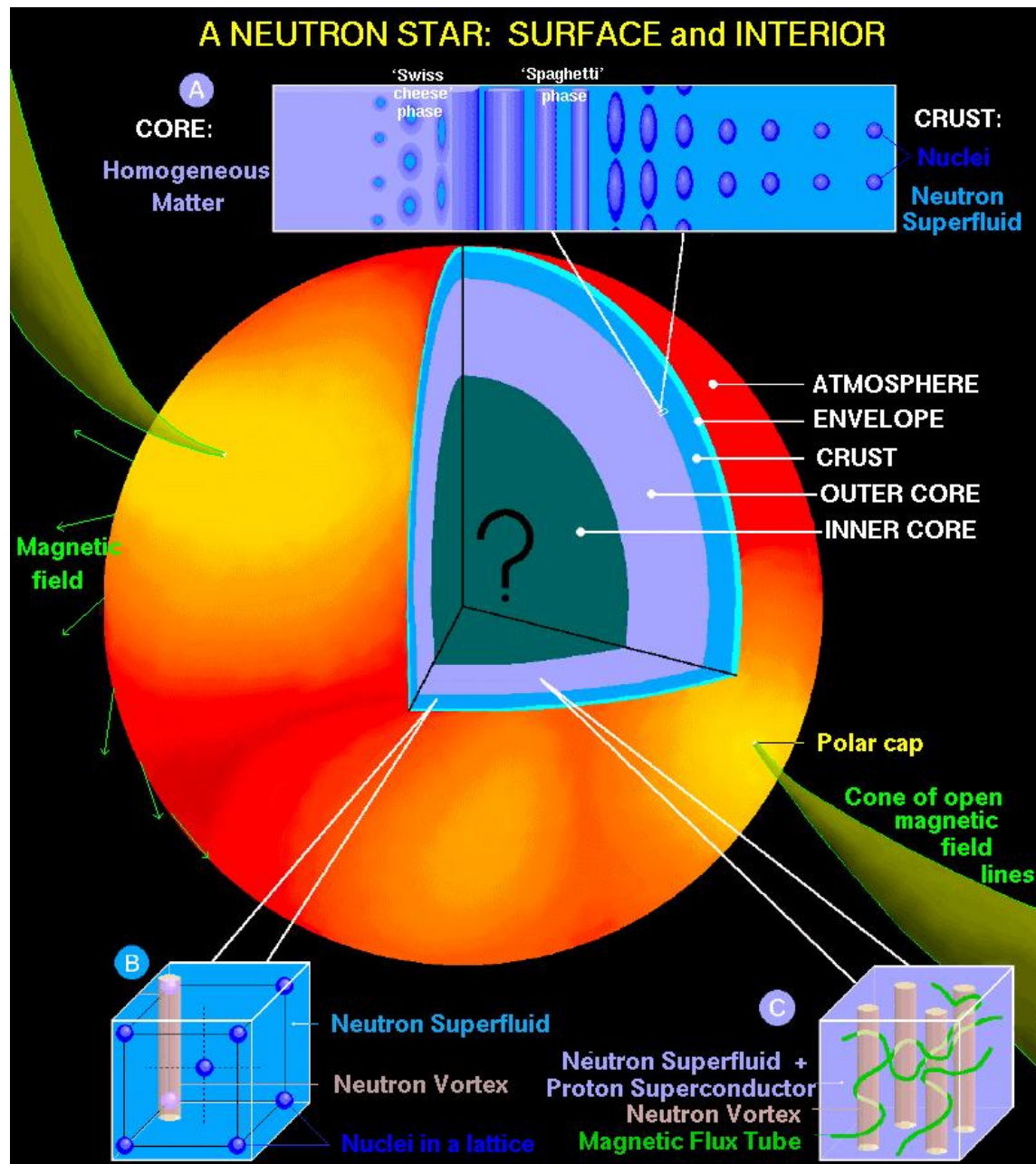
Quantities:

B Gauss

g_{\oplus}

$\frac{R_{\text{NS}}}{m_p}$

$\Phi \approx 10^{12}$ volts



D. Page

Superfluid vortices in liquid He

155

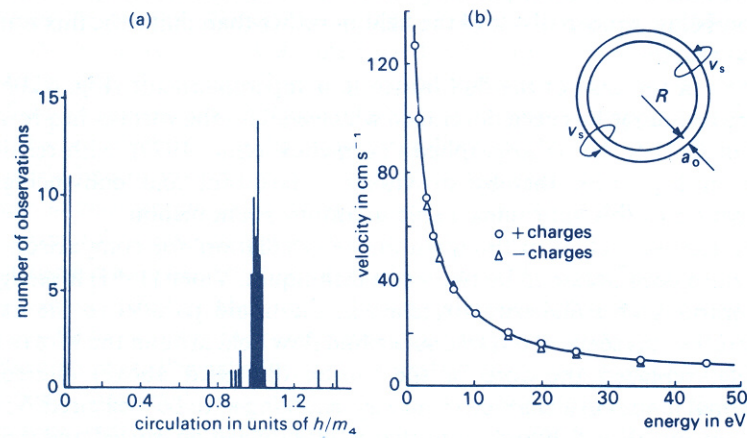


Figure 5.20 Determination of the quantum of circulation in HeII by two different methods. (a) Measurements of the circulation around a vibrating wire (Vinen, 1961) congregate predominantly about the value h/m_4 . (b) The calculated variation of velocity with energy (full curve) for a charged vortex ring (inset) can be fitted extremely accurately to the experimental measurements (points) on the assumption that the quantum of circulation is h/m_4 (Rayfield and Reif, 1964).

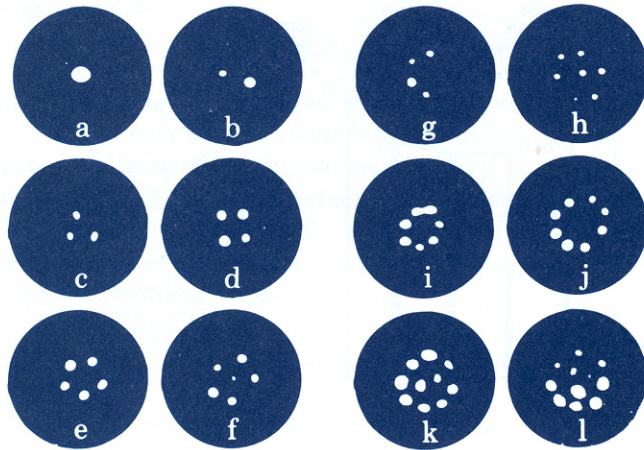


Figure 5.19 Stable (and metastable) vortex arrays in a rotating bucket of HeII for various angular velocities, photographed from above in the rotating frame of reference (Yarmchuk *et al.*, 1979). Each white spot corresponds to the presence of an individual quantized vortex line. The inner diameter of the bucket was 2 mm and its angular velocity varied between 0.3 and 0.9 rad s^{-1} . (Reproduced by courtesy of R.E. Packard.)

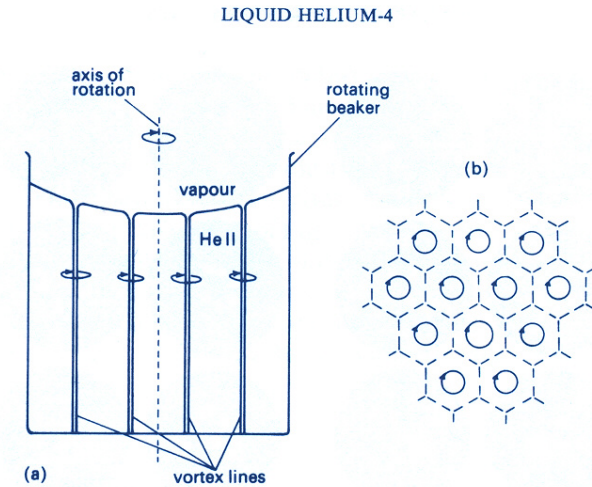


Figure 5.18 HeII in a rotating beaker. (a) Side view, with the vortex core diameters exaggerated for clarity. (b) Top view of a section of the surface: the dashed lines indicate points where the helium is at rest in a frame of reference rotating with the beaker; and the full arrows represent the superfluid velocity as viewed from the rotating frame.

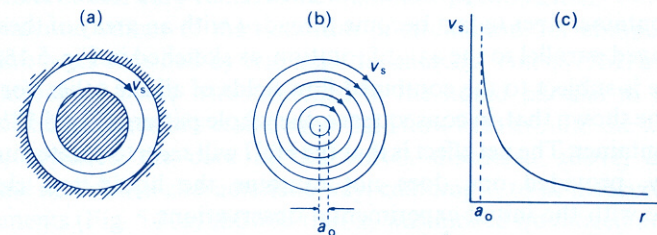


Figure 5.17 (a) Flow of the superfluid component with tangential velocity v_s around the annular space between two cylinders. (b) Sketch of a vortex line, with superfluid flowing around a core of radius a_0 at a velocity v_s , which depends inversely upon the radial distance r from the centre, as indicated in (c).

Neutron stars: Cooper pairing of neutrons

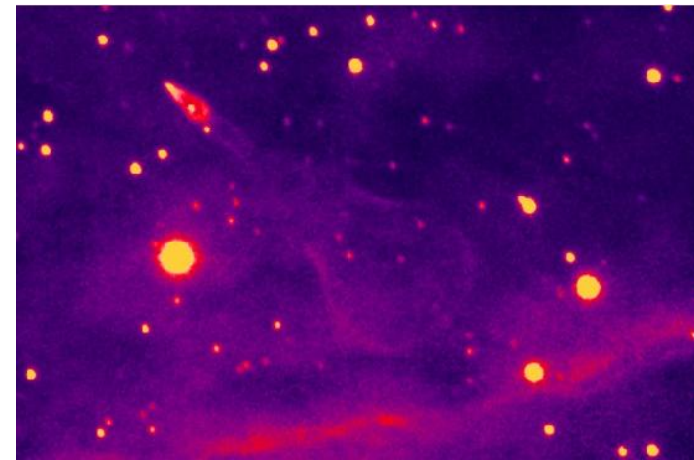
Bow Shocks

Guitar Nebula:

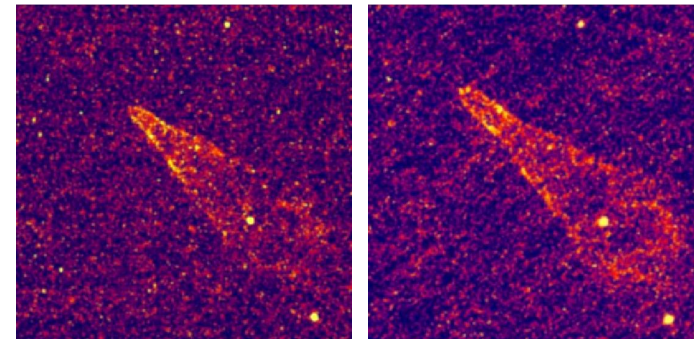
- Ordinary pulsar
 - $P = 0.68 \text{ s}$
 - $B = 2.6 \times 10^{12} \text{ G}$
 - $P/2\dot{P} = 1.1 \text{ Myr}$
 - $E = I \Omega \dot{\Omega} = 10^{33.1} \text{ erg s}^{-1}$
 - $D \sim 1.9 \text{ kpc}$ (from DM)
- 1600 km s^{-1} at nominal distance
- Will escape the Milky Way

Radius of curvature of bowshock nose increased from 1994 to 2001, corresponding to a 33% decrease in ISM density

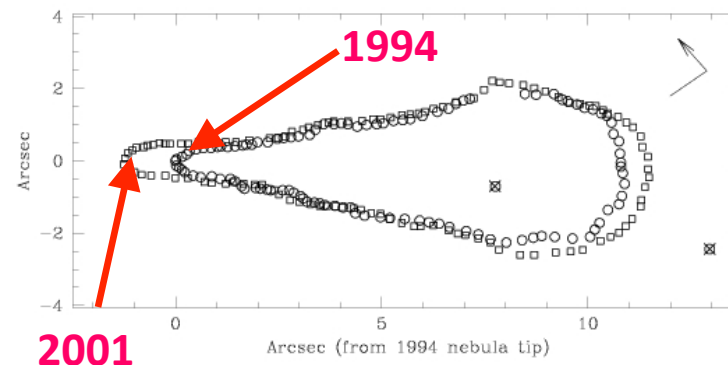
The pulsar is emerging from a region of enhanced density



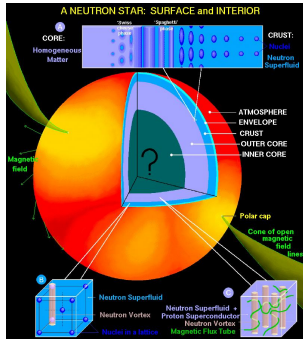
Palomar
H α image



HST
WFPC2
H α



Chatterjee & Cordes 2004

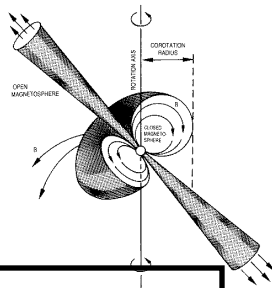


Differential rotation,
superfluid vortices

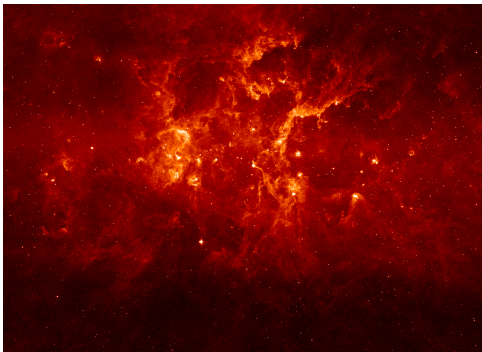
Pulsars Probe Everything!

Uncertainties in planetary
ephemerides and
propagation in
interplanetary medium

Glitches
Spin noise
Magnetosphere

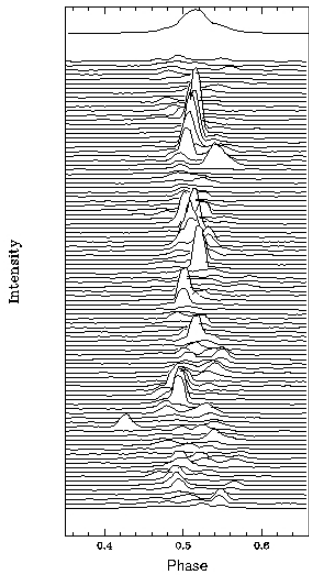


Interstellar dispersion
and scattering

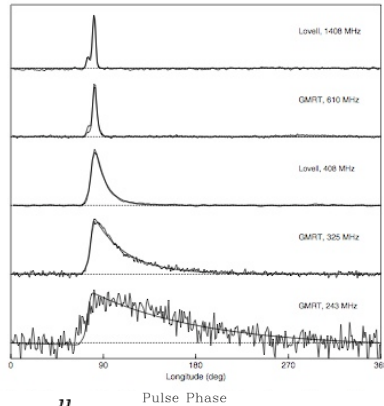
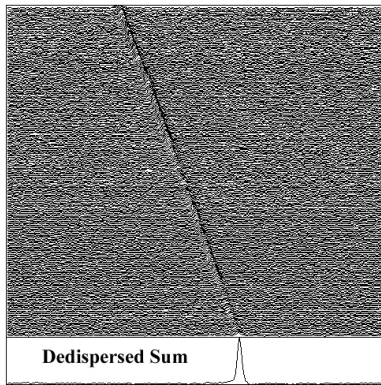


Emission
region:
beaming
and
motion

GPS time transfer
Additive noise
Instrumental
polarization



F
R
E
Q
U
E
N
C
Y



$$\text{TIME} \quad \text{DM} = \int_0^l n_e dl$$

12 Sep '11

Course II: Compact Objects, the Dynamic Sky and 21st Century R

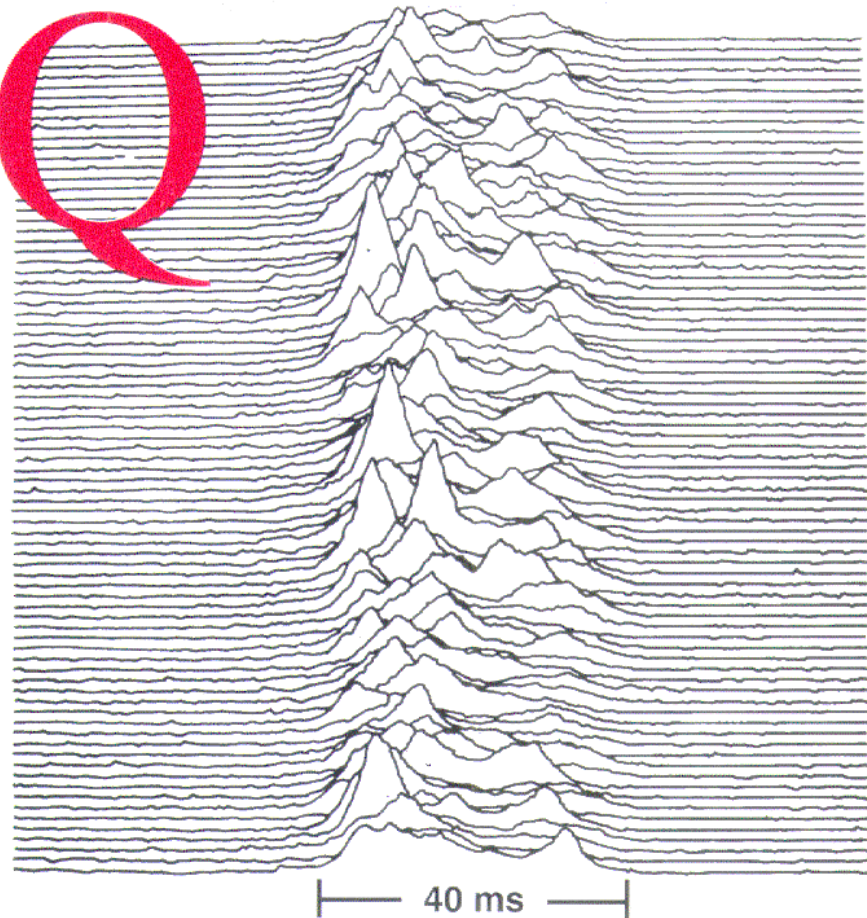
AA2JQ

Hal Craft

3 Sunny Slope Road
Ithaca, New York
14850



Pulses from Pulsar PSR 1919+21 at 318 MHz



By HDC @ the Arecibo Observatory, Summer 1968

Album cover
for 1979
punk rock
group



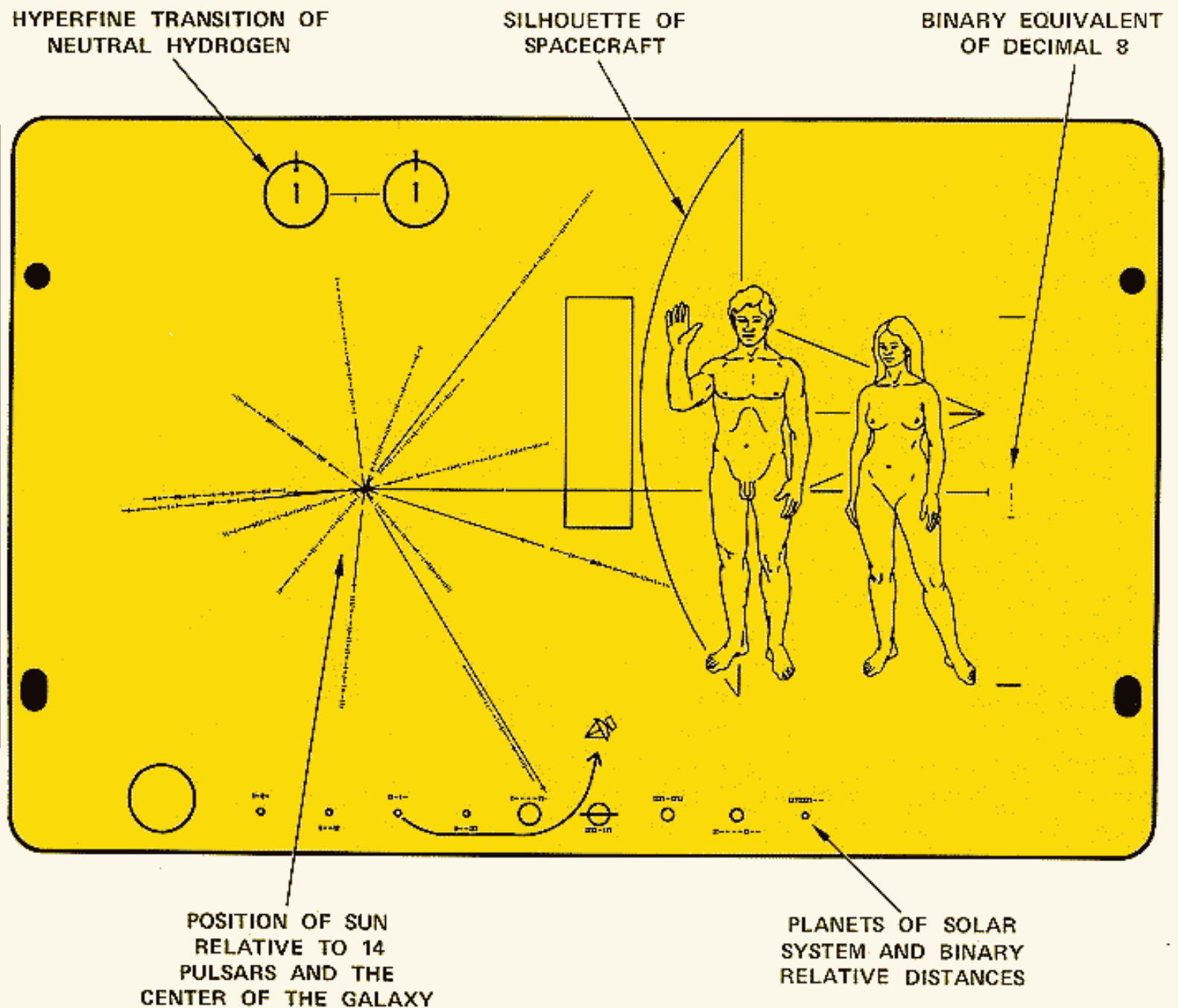
Plaque on
the Pioneer
10,11
spacecraft

Launched in
1972,
communicat
ed with until
2003, 1995

Escaping
the solar
system



12 Sep '11



Definitions and Units

- $\lambda\nu = c$
 - wavelength λ : meters, centimeters
 - frequency ν : MHz and GHz
- Flux density:
 - 1 Jansky = 10^{-23} erg s⁻¹ cm⁻² Hz⁻¹ = 10^{-26} watts m⁻² Hz⁻¹
- Angular size:
 - 1 milli-arcsecond = 10^{-3} arc sec (requires very long baseline interferometry)
 - 0.1 to 1 deg = typical field of view with a single dish antenna (FoV $\sim \lambda/\text{diameter}$)
- Solid angle: 4π steradians = 41,253 deg² in the entire sky
- Pulsar spin periods: 1.4 milliseconds to ~ 10 seconds
- Magnetic fields (1 G = 1 Gauss = 10^{-4} Tesla)
 - 1 G \sim magnetic field of the Earth
 - 10^{12} G \sim surface field in a “canonical” pulsar
 - $>10^{14}$ G \sim magnetic field of a “magnetar”
 - 10^9 G \sim magnetic field of a millisecond pulsar
 - 10^{-6} G \sim magnetic field in the interstellar medium

Basic Lessons: A Minimal Set

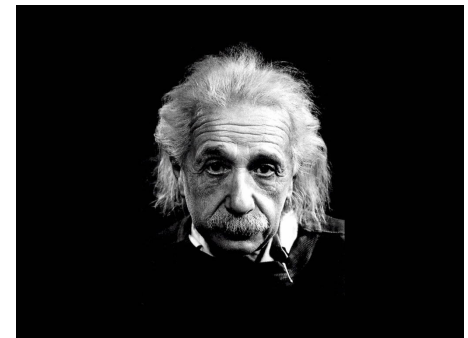
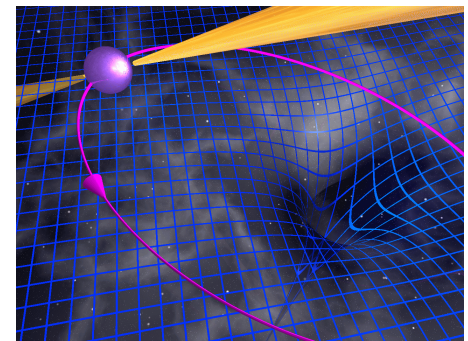
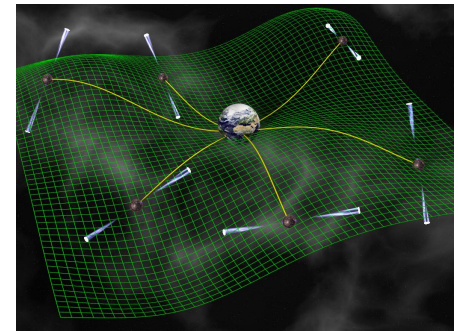
- Phase, phase, phase [e.g. in electric field $\sim \cos(\omega t + \text{phase})$]
 - Imaging, polarization, dedispersion, radar
 - Phase restoration w.r.t. propagation and instrumentation
- Noise, noise, noise
 - All celestial signals are statistical in nature
 - High photon number \Rightarrow Waves & fields, not photons; and Gaussian statistics (mostly)
 - Radiometer equation
- Radiation: thermal, nonthermal, incoherent, coherent
- Matched filtering: the universal tool for detecting signals
- Fourier transforms: If something varies, Fourier transform it !
- Radio frequency interference (RFI): a low/high tech nuisance needing high-tech solutions
- Array telescopes: the future
- Computational astronomy and data mining (“cyberinfrastructure”)
 - Massive data sets
 - Virtual Observatories

Black Belt Puzzle

- Can individual radio photons be detected?
(note single optical, X-ray, gamma-ray photons are detected)

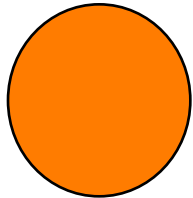
Current Main Areas of Pulsar Science

- **Nano-Hz gravitational waves**
 - Timing of 10^x millisecond pulsars (MSPs) at < 100 ns precision
 - 5 to 10 years needed for detection of GW background(s)
 - Longer program to characterize spectrum, detect single sources
- **Testing GR and other ToGs**
 - Precision timing of relativistic binaries
 - NS+NS, NS+BH, NS+WD
 - Precision masses of NS (incl. $2M_{\odot}$)
 - Pulsars orbiting the $4 \times 10^6 M_{\odot}$ black hole in Galactic center
- **Significant additional science:**
 - Multi-messenger studies
 - Binary mergers \rightarrow GRBs, GWs, radio bursts?
 - Transients (discovery space)
 - ISM studies (WIM)
 - needed for precision timing (dispersion, multipath)
 - Emission mechanism studies
 - fluctuations relevant to precision timing (pulse jitter)
 - multi-wavelength studies (especially radio + gamma rays with Fermi gamma-ray satellite)



Endstates of Stellar Evolution

Main sequence star

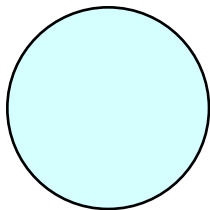


0.1 to 8 M_{sun}

Compact Remnant

White dwarf 0.1 to $\sim 1.2 M_{\text{sun}}$

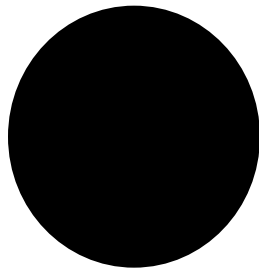
Degenerate electron pressure



8 to 20 (?) M_{sun}

Neutron star 1.3 to $< 3 M_{\text{sun}}$

Degenerate neutron pressure



$> 20 M_{\text{sun}}$

Black hole $> 3 M_{\text{sun}}$

Gravity wins

Complications: mass exchange in binary systems

Neutron Stars

Background:

- 1931: understanding of white dwarfs (Chandrasekhar)
- 1932: neutron discovered (Chadwick)
- 1934: neutron stars (Baade & Zwicky)
- 1939: first models (Oppenheimer & Volkoff)

Detectable?

- 1967: Thermal radiation (10^6 K, 10 km) \Rightarrow bleak
Radio pulsars (serendipitous)
Gamma-ray bursts (ditto)
- 1968: Pulsar discovery announced
Crab pulsar discovered
- 1969: Crab pulsar spindown measured
& clinched the NS hypothesis (T. Gold)

“With all reserve we advance the view that supernovae represent the transition from ordinary stars into neutron stars, which in their final stages consist of closely packed neutrons.”
Baade & Zwicky, 1934, Phys Rev, 45, 138

THE MAXIMUM MASS OF IDEAL WHITE DWARFS

By S. CHANDRASEKHAR

ABSTRACT

The theory of the *polytropic gas spheres* in conjunction with the equation of state of a *relativistically degenerate electron-gas* leads to a *unique value for the mass of a star* built on this model. This mass ($=0.91\odot$) is interpreted as representing the upper limit to the mass of an ideal white dwarf.

In a paper appearing in the *Philosophical Magazine*,¹ the author has considered the density of white dwarfs from the point of view of the theory of the polytropic gas spheres, in conjunction with the degenerate non-relativistic form of the Fermi-Dirac statistics. The expression obtained for the density was

$$\rho = 2.162 \times 10^6 \times \left(\frac{M}{\odot} \right)^2, \quad (1)$$

where M/\odot equals the mass of the star in units of the sun. This formula was found to give a much better agreement with facts than the theory of E. C. Stoner,² based also on Fermi-Dirac statistics but on uniform distribution of density in the star which is not quite justifiable.

In this note it is proposed to inquire as to what we are able to get when we use the relativistic form of the Fermi-Dirac statistics for the degenerate case (an approximation applicable if the number of electrons per cubic centimeter is $> 6 \times 10^{29}$). The pressure of such a gas is given by (which can be shown to be rigorously true)

$$P = \frac{1}{8} \left(\frac{3}{\pi} \right)^{\frac{1}{3}} \cdot hc \cdot n^{4/3}, \quad (2)$$

where h equals Planck's constant, c equals velocity of light; and as

$$n = \frac{\rho}{\mu H(1+f)}, \quad (3)$$

¹ 11, No. 70, 592, 1931.

² *Philosophical Magazine*, 7, 63, 1929.

Degenerate electron
pressure balances
gravity:

Pauli exclusion principle:

2 electrons/state

Uncertainty Principle:

$$\Delta \vec{x} \Delta \vec{p} > \hbar$$

Compression of star
decreases Δx and increases
 $\Delta p \Rightarrow$ pressure

Maximum mass when
electrons become relativistic
and pressure increases too
slowly to counteract gravity

LETTERS TO THE EDITOR

ASTRONOMY

Energy Emission from a Neutron Star

ALTHOUGH there are still many problems concerning the supernovae, there is little doubt that a very dense stellar core has to be left behind after the explosion (at least in some cases). During the contraction of this core, inverse β reactions take place and transform most of the nuclei and electrons into neutrons. If the mass of the neutron star does not exceed a critical value of about one or two solar masses, a stable equilibrium situation can be reached with the gas pressure balancing the gravitational force.

A newly formed neutron star is an excited object. Apart from its thermal content (which will be dissipated very fast because of neutrino processes), there will also be much energy stored in vibrational and rotational form. The problem therefore arises of finding out whether the energy stored in the neutron star plays an important part in connexion with the activity observed in some supernova remnants such as the Crab Nebula.

The vibrations of the neutron star, however, do not last long enough for our purposes. The principal reason for this is that the emission of gravitational waves will damp quadrupole and higher order pulsations in a few seconds (ref. 1 and unpublished work of T. A. Wheeler and A. Zoe). Moreover, because the stellar rotation will mix the radial modes of vibrations with the non-radial one, all the vibrations are going to disappear very quickly.

It seems more rewarding therefore to look for some mechanisms by which the neutron star can release either its magnetic or its rotational energy or both. In this communication I would like to outline the principal features of a possible model of this kind.

The existence of very strong magnetic fields in the neutron stars has been suggested as a consequence of the compression of an ordinary stellar field. The underlining assumption here is that the conductivity, σ , of the stellar matter is so high that the decay time for the field exceeds the collapse time. As the collapse time is of the order of seconds, this means

$$\tau_{\text{decay}} \sim \frac{4\pi\sigma R^2}{c^2} \gg 1 \quad (1)$$

where R is the radius of the neutron star (about 10^6 cm). For any conceivable value of σ this is a very weak requirement. We can therefore expect the field strength to increase as $1/R^2$ during the contraction so that fields as high as 10^{10} – 10^{14} gauss can be produced².

If we assume that the magnetic field is that of a dipole, the angle between the dipole and the angular momentum is likely to be arbitrary (oblique rotator). Actually, even if the two axes coincided in the pre-supernova star, the mass loss occurring during the explosion is unlikely to be perfectly symmetric, especially in the presence of strong magnetic fields. The mutual inclination of the two axes will then be modified and both the rotation and the magnetic field will tend to flatten the star (but along different directions). The shape of the star is going to be rather complicated and the body will rotate about an axis which is not a principal axis of inertia. This is a non-equilibrium situation and the motion will be such that the total angular momentum is constant while the

instantaneous axis of rotation precesses. Stresses and magneto-hydrodynamic waves are therefore to be expected at the surface of such a star³. This will dissipate energy with the final result of bringing the system into an equilibrium state, that is, the magnetic axis in coincidence with the axis of rotation. Acceleration of particles to relativistic energies is to be expected under these circumstances.

The same picture of an oblique rotator leads also to a different possibility, that is, that the neutron star might directly emit electromagnetic waves of very low frequency (in the kc/s range). This idea has been suggested by Hoyle, Narlikar and Wheeler⁴ as a possible consequence of the vibrations of a magnetic neutron star. Because the rapid damping of the vibrations makes it difficult to retain this suggestion in the original form, I wish to point out that the oblique rotator model also results into an analogous emission of electromagnetic waves.

If d_0 is the projection of the dipolar moment on the plane perpendicular to the axis of rotation and Ω is the angular velocity of the star, there will be a monochromatic emission of electromagnetic waves at the frequency $\omega = \Omega$. The corresponding intensity is given⁵

$$I = \frac{2}{3} \frac{d_0^2 \Omega^4}{c^3} \quad (2)$$

If we take d_0 to be about $H_0 R^2 = 10^{10} \times 10^{18}$ gauss cm² and $\Omega = 10^4$ sec⁻¹, the intensity would be of the order of 2×10^{49} ergs/sec.

We must, however, ask ourselves the question whether this radiation will ever arise. Any variation of the magnetic field can actually be compensated by the electric currents in the surrounding matter. Hoyle, Narlikar and Wheeler⁴ have stated that the strong gravitational field creates a near vacuum immediately outside the star so that in this region no propagation difficulty would arise. This equilibrium picture, however, seems unlikely for a newly formed neutron star, soon after the supernova explosion. It is then necessary to evaluate the maximum gas density which still allows the emission of electromagnetic waves. This is easily done if we note that the maximum current density in the gas is given by $j = n_e e c$, where n_e is the electron density. The Maxwell equation

$$\text{curl } \vec{H} = \frac{4\pi}{c} \vec{j} \quad (3)$$

gives then the maximum induced field. If we take $\text{curl} \sim 1/r$ (r is a characteristic length of the order of the size of the system) we obtain

$$H_{\text{max}} \sim 4\pi n_e e r \quad (4)$$

Any variation of the magnetic field of the order of the field itself cannot therefore be compensated by the induced currents if

$$n_e < \frac{H}{4\pi e r} \quad (5)$$

Assuming again H to be about 10^{10} gauss and $r = R$, about 10^6 cm, we must require $n_e < 10^{13}$ cm⁻³. This limit is certainly not very stringent and condition (5) is likely to be violated only at the beginning, that is, soon after the birth of the neutron star.

Once the electromagnetic waves are emitted and propagate in the supernova remnant, they will be reflected by the circumstellar gas if the plasma frequency exceeds the radiation frequency. For $\omega = 10^4$ sec⁻¹ this happens if $n_e > 2.5 \times 10^{-3}$ cm⁻³ which is now a very low figure. The electromagnetic waves would therefore be unable to reach us, but by this means a large amount of energy and momentum could be pumped from the neutron star into the supernova remnant. In particular, the radiation will give an outward momentum to the nebula by being reflected and therefore accelerate its expansion. As a matter of fact, there is observational evidence that the motion of the Crab Nebula has been accelerated after the

A prescient letter to
Nature by Franco Pacini,
1967

Proposes energy losses
from NS by magnetic
dipole radiation

Observation of a Rapidly Pulsating Radio Source

by

A. HEWISH
S. J. BELL
J. D. H. PILKINGTON
P. F. SCOTT
R. A. COLLINS

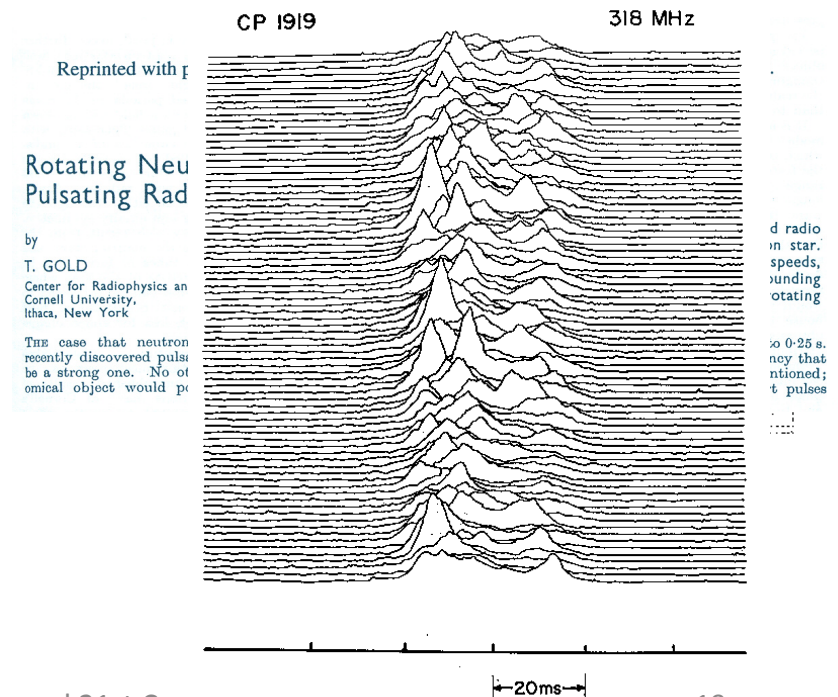
Mullard Radio Astronomy Observatory,
Cavendish Laboratory,
University of Cambridge

Unusual signals from pulsating radio sources have been recorded at the Mullard Radio Astronomy Observatory. The radiation seems to come from local objects within the galaxy, and may be associated with oscillations of white dwarf or neutron stars.



Initial Ideas About Radio Pulsations

- LGMs 1, 2, 3 and 4
- White dwarf oscillations
- Orbital motion of white dwarfs
- Neutron star spins
 - Electromagnetic radiation at the spin frequency
 - Lighthouse model (spinning beam): T. Gold
- Similar richness of interpretations for Gamma-ray bursts up to 1990s

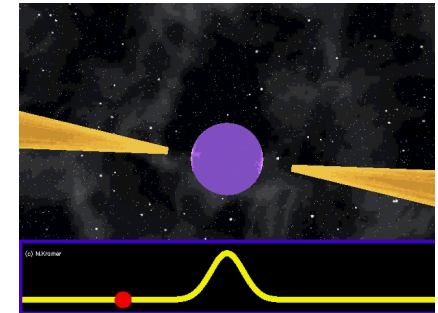


Pulsar Sounds

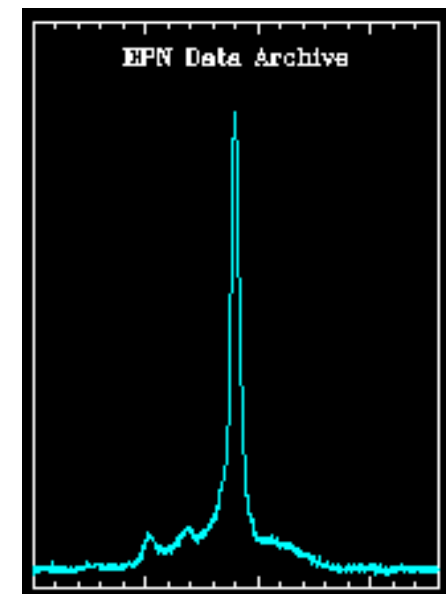
Radio signals demodulated into audio signals

Pulsar	P (ms)	$f=1/P$ (Hz)
B0329+54	714	1.4
B0950+08	253	3.9
B0833-45 (Vela)	89	11.2
B0531+21 (Crab)	33	30.2
J0437-4715	5.7	174
B1937+21	1.56	642

sound file



**J0437-4715
profile**

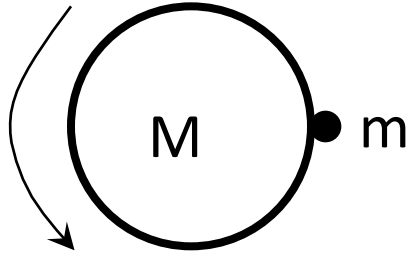


Stability of Spinning Objects

How large can Ω be while keeping m gravitationally bound to M ?

$$\Omega = \frac{2\pi}{P}$$

or



How dense does the star have to be for the star to remain bound at spin period P ?

Need gravity $>$ centrifugal force:

First pulsar CP1919+21 (1967):
 $P = 1.33 \text{ s} \Rightarrow \bar{\rho} > 10^{5.9} \text{ gm cm}^{-3}$

$$\frac{GMm}{R^2} > \frac{mV^2}{R} = \frac{m(\Omega R)^2}{R}$$

Ordinary star: $\bar{\rho} \sim 1 \text{ gm cm}^{-3}$

$$\frac{GM}{R^3} > \left(\frac{2\pi}{P}\right)^2$$

White dwarf: $\bar{\rho} \sim 10^6 \text{ gm cm}^{-3}$

$$\frac{GM}{4\pi R^3/3} > \frac{3\pi}{P^2}$$

Crab pulsar (1968):
 $P = 33 \text{ ms} \Rightarrow \bar{\rho} > 10^{11} \text{ gm cm}^{-3}$

$$G\bar{\rho} > \frac{3\pi}{P^2} \Rightarrow \bar{\rho} > \frac{3\pi}{GP^2}$$

Millisecond pulsar (1982):
 $P = 1.5 \text{ ms} \Rightarrow \bar{\rho} > 10^{13.8} \text{ gm cm}^{-3}$

\therefore Pulsars must have $>$ nuclear density \Rightarrow neutron stars

Spindown

- The rotating NS hypothesis implies that periods should lengthen, i.e. the stars should slow down in their spin (T. Gold)
- The Crab pulsar (1968) was observed to slow down, thus clinching the rotating NS hypothesis
- Energy loss rate:

$$\dot{E} = \frac{d}{dt} \left(\frac{1}{2} I \Omega^2 \right) = I \Omega \dot{\Omega}$$

$$I \approx 10^{45} \text{ gm cm}^2$$

$$\dot{E} \approx 10^{31.6} P^{-3} \dot{P}_{-15} \text{ erg s}^{-1}$$

Magnetic Dipole Radiation

$$\dot{E} = \frac{2}{3c^3} |m|^2 \Omega^4 \sin^2 \alpha$$

m = magnetic moment

$$\dot{\Omega} \propto -\Omega^n = -\left(\frac{2m^2 \sin^2 \alpha}{3c^3 I}\right) \Omega^3$$

n = braking index

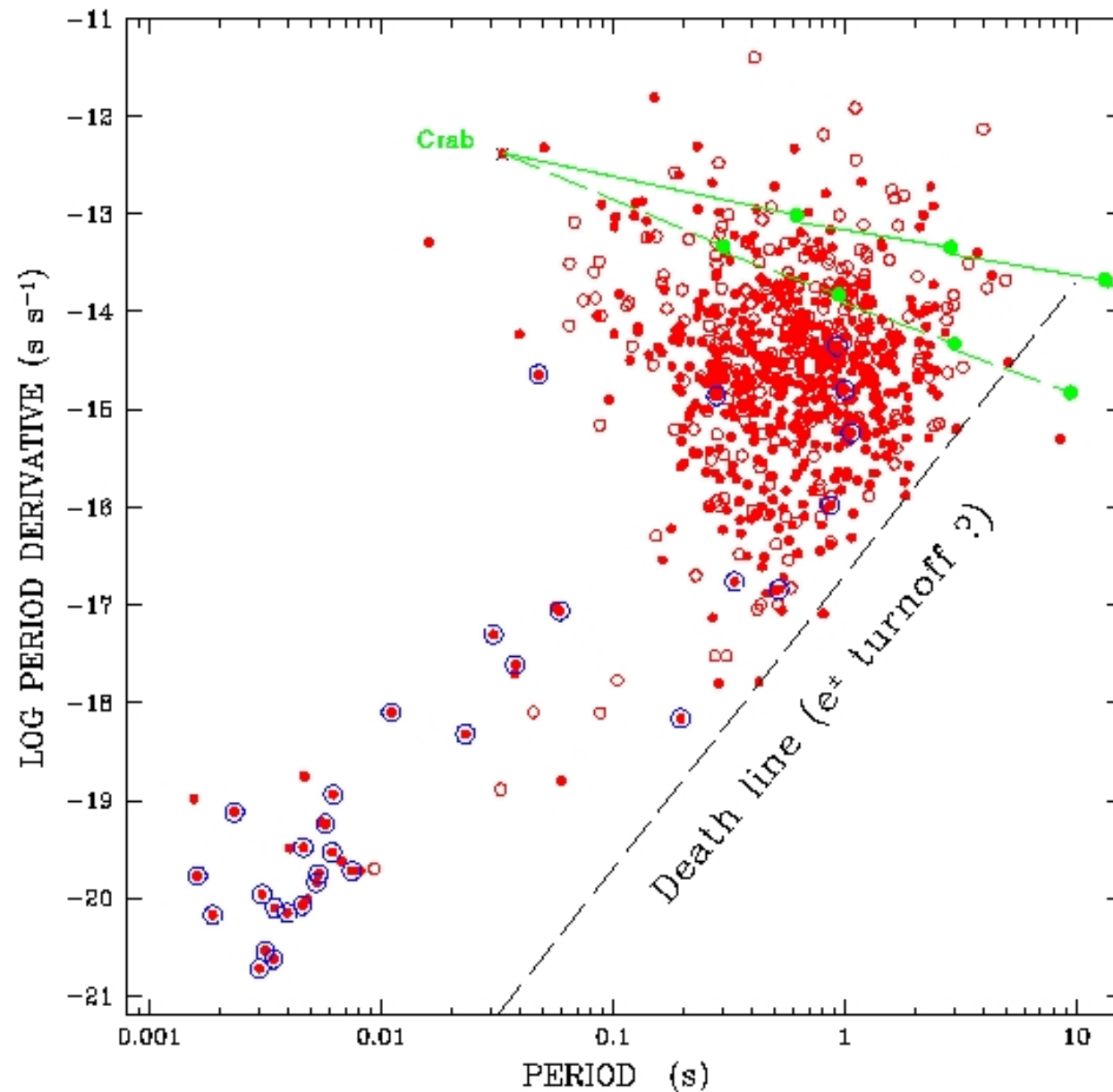
$$B = \frac{|m|}{R^3} = \left(\frac{3c^2 I}{8\pi^2 R^6 \sin^2 \alpha}\right)^{1/2} \sqrt{P\dot{P}}$$

$$= 3.2 \times 10^{19} \sqrt{P\dot{P}} \text{ Gauss}$$

$$= 1.0 \times 10^{12} \sqrt{P\dot{P}_{-15}} \text{ Gauss}$$

$$\tau = \frac{1}{n-1} \frac{P}{\dot{P}} = \frac{P}{2\dot{P}} \text{ (if } n = 3\text{)} \quad \text{Characteristic age}$$

P - PDOT DIAGRAM 950 pulsars



Spindown

$$\dot{P} \propto P^{-0.5}, n = 2.5$$

$$\dot{P} \propto P^{2-n} = P^{-1}$$

Spindown and Glitches

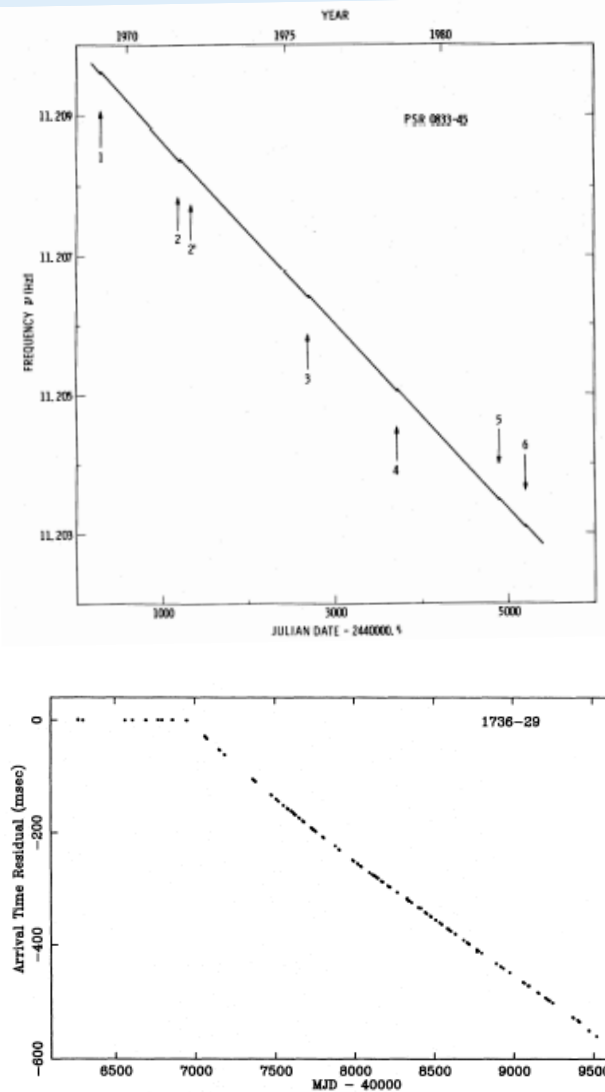
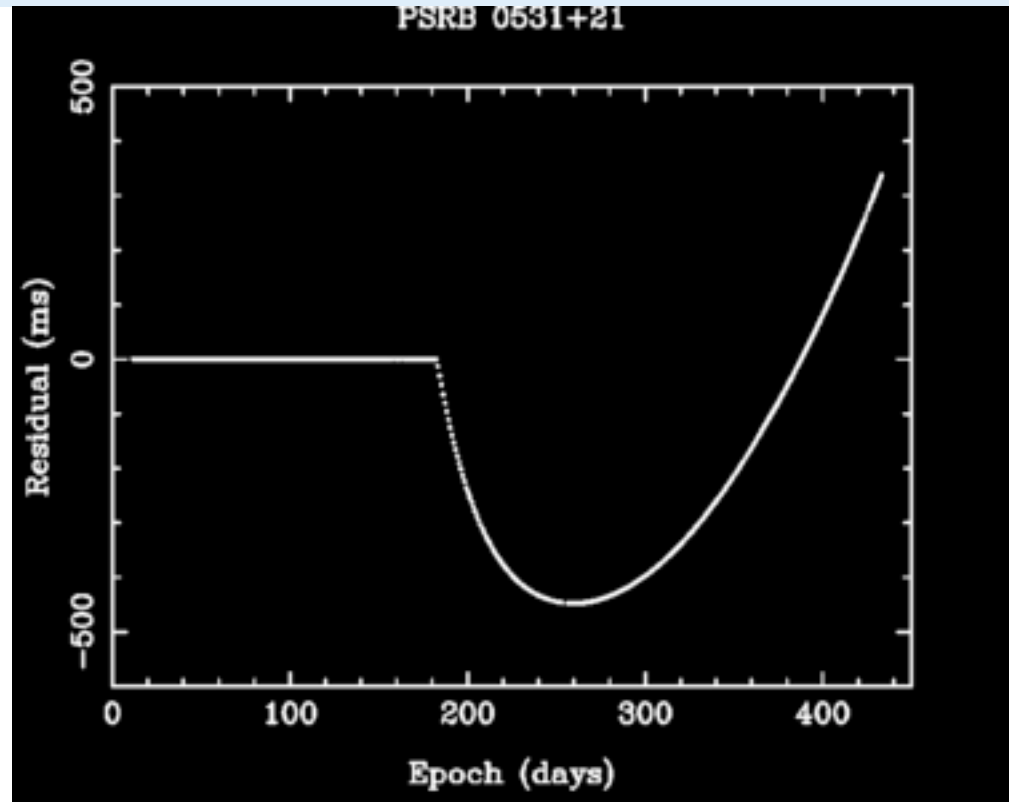
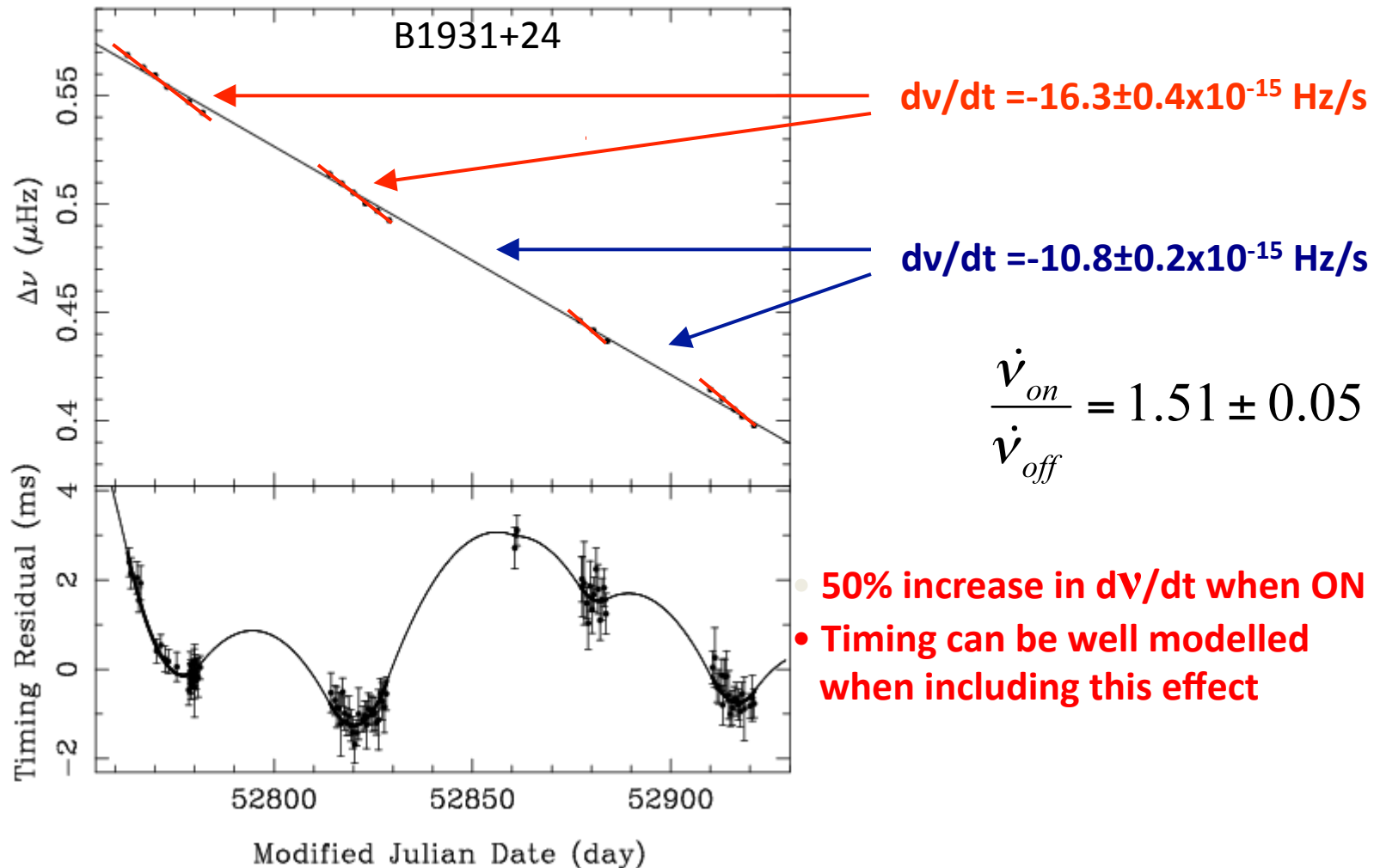


Figure 1. Timing residuals showing the glitch in PSR B1736 - 29.



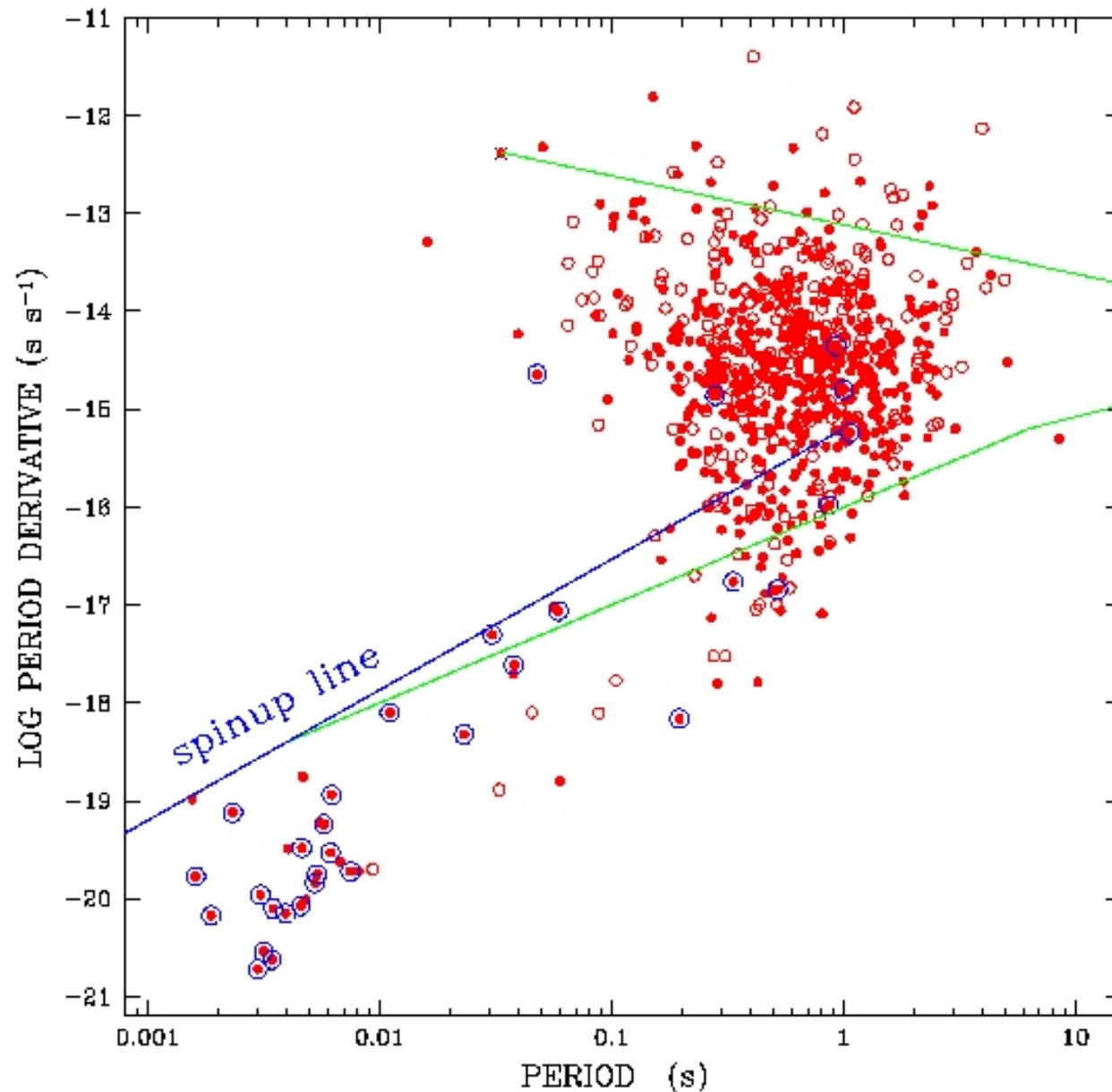
Intermittent Torques and Radio Emission

Kramer et al. 2006



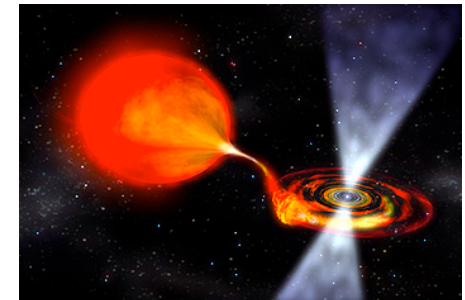
...the spin-down is faster when radio emission is on

P - PDOT DIAGRAM 950 pulsars



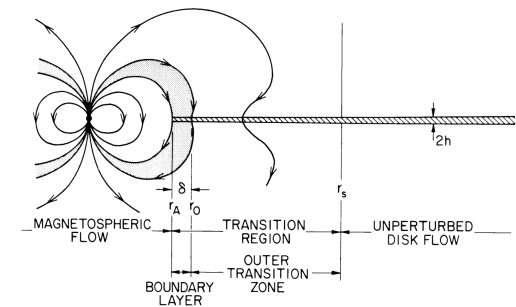
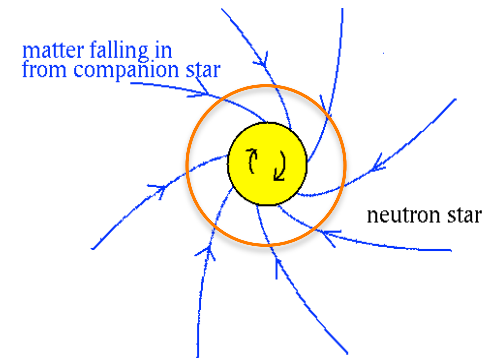
Spinup:

Accretion
“recycles” the
pulsar by spinning
it up back across
the death line and
it attenuates the
magnetic field



Recycling: Spinup in Accreting Systems

- Accreting material carries angular momentum that spins up the NS
- Spinup stops when the corotation velocity = Keplerian velocity at the Alfven radius
- Think of a top or gyroscope that you spin up by pulling a string
 - The spinup stops when the velocity of the gyroscope equals that of the string
- Equilibrium period:



Ghosh & Lamb (1978)

$$P_{\text{eq}} \approx 0.93 \text{ s} \left(\frac{\dot{P}}{10^{-15} \text{ s s}^{-1}} \right)^{3/4}$$

Details of Equilibrium Period Calculation

Equilibrium Period

$v_{\text{spin}} = v_{\text{Kepler}}$ at Alfvén radius

$$\Omega r_A = \frac{2\pi}{P} = \left(\frac{GM}{r_A^3} \right)^{1/2}$$

Calculate r_A using:

$$B^2/8\pi = \rho v^2$$

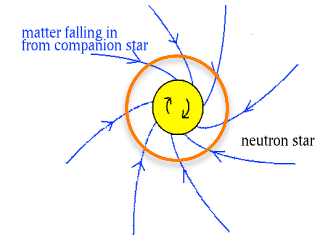
$$B = B_0(R/r)^3 \text{ (dipole)}$$

$$\Rightarrow r_A \sim \left(\frac{B_0^2 R^6}{\dot{M} \sqrt{GM}} \right)^{2/7}$$

Relate B_0 to P, \dot{P} assuming magnetic dipole radiation:

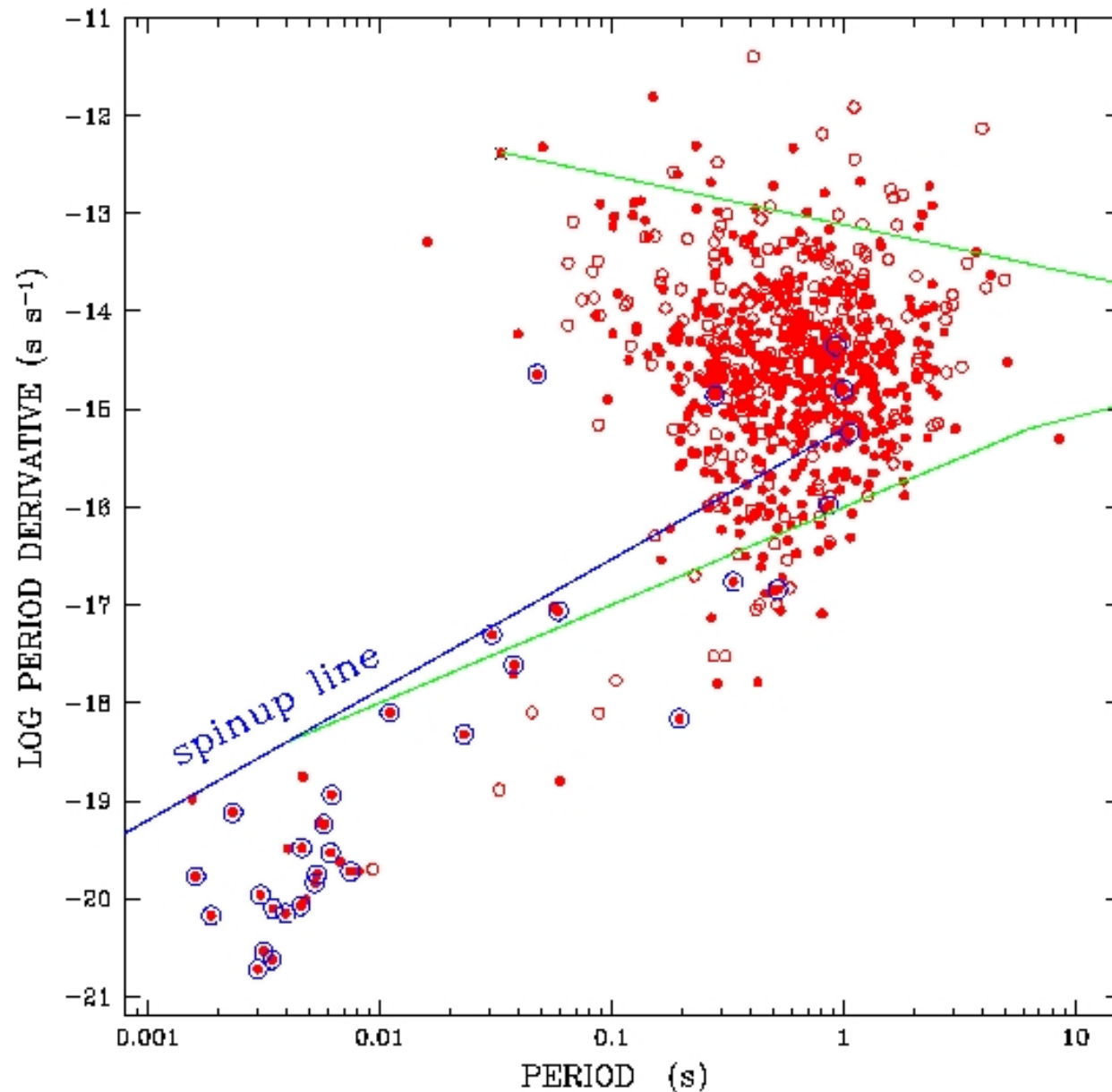
$$B_0^2 \propto P \dot{P}$$

$$\Rightarrow P_{\text{eq}} \propto \dot{P}^{3/4}$$



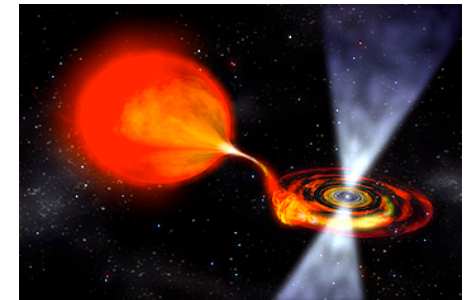
Alfvén radius =
radius where
magnetic energy
density \sim kinetic
energy density

P - PDOT DIAGRAM 950 pulsars



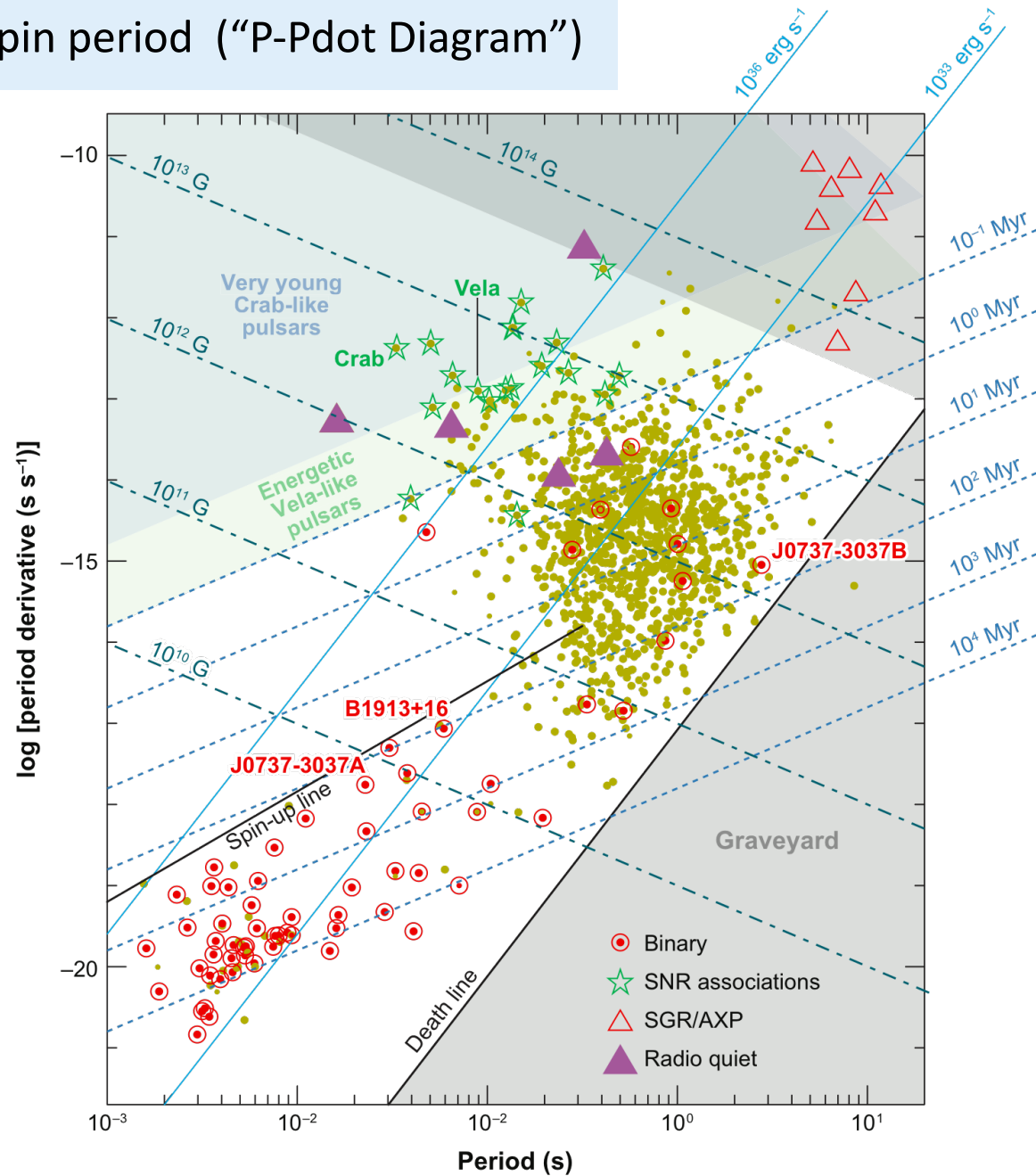
Spinup:

Accretion
“recycles” the
pulsar by spinning
it up back across
the death line and
it attenuates the
magnetic field



Spin period derivative vs. Spin period (“P-Pdot Diagram”)

Kramer and Stairs 2008



Pulsar Populations: $P - \dot{P}$ Diagram

- **Magnetars+high-field pulsars**

- $P \sim 5-12$ s
- $B \sim 10^{14} - 10^{15}$ G

- **Canonical pulsars**

- $P \sim 20\text{ms} - 5\text{s}$
- $B \sim 10^{12 \pm 1}$ G

- **Recycled/Millisecond pulsars (NS-NS binaries, MSPs)**

- $P \sim 1.4 - 20\text{ms}$
- $B \sim 10^8 - 10^9$ G

- **Braking index n :**

- $\dot{P} \propto P^{2-n}$, $n=3$ magnetic dipole radiation

- **Death line**

- **Strong selection effects**

