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Can a background of gravitational waves constrain the star formation history of the universe?

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Abstract

In general, the history of star formation of the universe is obtained from hydrodynamical simulations in a Λ-CDM cosmology. On the other hand, a complete study of the formation of the first objects and the feedback effects on the ambient medium produced by them, is not still possible with the present numerical approaches. However, formation of stars at different redshifts can produce a stochastic background of gravitational waves in the VIRGO and LIGO frequency band and so the detection of this background could directly be used to obtain information about the star formation rate density of the Universe. Here, we use the Press–Schechter formalism to calculate the co-moving abundance of halos and the mass contained within the collapsed objects of a given mass range. Then, we study, in particular for a pair of advanced LIGO observatories, what limits could be imposed on the fraction of baryons converted into stars within halos that collapse at redshifts $5 \leqslant z \leqslant 30$.

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1. Introduction

Because of the fact that gravitational waves (GWs) are produced by a large variety of astrophysical sources and cosmological phenomena, it is quite probable that the Universe is pervaded by a background of such waves. Population III stars, phase transitions in the early Universe and cosmic strings and a variety of binary stars are examples of sources that could produce such a putative background of GWs (see, e.g., Blair and Ju (1996), Ferrari *et al* (1999a, 1999b), Schutz (1999), Giovannini (2000), Maggiore (2000), among others).

On the other hand, concerning the star formation at high redshift, our knowledge is mainly based on numerical simulations performed by hydrodynamical codes in a Λ-CDM cosmology. Certainly, these simulations must reproduce the observable Universe at redshifts $z < 5$.

The evidence for the existence of a large star formation at high redshift comes from, among others, the Gunn–Peterson effect (Gunn and Peterson 1965) and from the metallicity of $\sim 10^{-2} Z_{\odot}$ found in high-z Lyα forest clouds (Songaila and Cowie 1996, Ellison *et al* 2000). These results are consistent with a stellar population formed at $z > 5$ (Venkatesan 2000).

Our objective in the present paper is show how a detection of a background of gravitational waves could be used to give us some insight into the star formation rate of the Universe. This kind of study could be also used to constrain the fraction of massive stars that generates black holes at high redshift.

The paper is organized as follows. In section 2, we present the Press–Schechter formalism used to determine the co-moving abundance of collapsed dark matter halos of a given mass. In section 3, we briefly review how to calculate the background of GWs produced during the formation of the stellar black holes (the reader will find a more detailed description in de Araujo *et al* (2002)). In section 4, we present some numerical results, the discussions and we consider the detectability of this putative GW background. Finally, in section 5 we present our conclusions.

2. The Press–Schechter formalism

According to the cold dark matter scenario for galaxy formation, galaxies and larger scale structures of the Universe are built up by the process of hierarchical clustering. An analytical formalism for the process of structure formation was developed by Press and Schechter in 1974. This formalism has been used by many authors in the study of the star formation rate at different redshifts (e.g., Steidel *et al* 1999).

The total mass contained per unit of co-moving volume within collapsed objects of logarithmic mass range $[\ln M, \ln(M + \mathrm{d}M)]$ is

$$\rho_{\mathrm{M}}^{\mathrm{halo}}(z)\,\mathrm{d}\ln M = -\left(\sqrt{\frac{\pi}{2}}\mathrm{e}^{-\nu^2/2}\right)\rho_0 \frac{\mathrm{d}\ln\sigma(M)}{\mathrm{d}\ln M}\nu(z, M)\,\mathrm{d}\ln M, \tag{1}$$

where ρ_0 is the mean co-moving density of the Universe and the other quantities are defined as

$$\sigma^2(R) = \int_0^{\infty} \frac{\mathrm{d}k}{k}\left[\frac{3(\sin kR - kR\cos kR)}{k^3R^3}\right]^2\left[\frac{k^3 P_{\mathrm{DM}}(k)}{2\pi^2}\right], \tag{2}$$

$$\nu(z, R) = \frac{\delta_{\mathrm{c}}}{D(z)\sigma(R)}, \tag{3}$$

with $M = 4\pi^3/3R^3\rho_0$.

The expressions for the normalized dark matter power spectrum $P_{\mathrm{DM}}(k)$ and the growth factor density, $D(z)$, are obtained from Padmanabhan (1993). In the above equation, δ_{c} is the critical density ($\delta_{\mathrm{c}} = 1.69$ for $\Omega_{\mathrm{m}} = 1$).

The total density parameter $\Omega_{\mathrm{M}}^{\mathrm{halo}}(z)$ of the matter (baryonic and dark matter) in the collapsed halos is

$$\Omega_{\mathrm{M}}^{\mathrm{halo}}(z) = \frac{\rho_{\mathrm{M}}^{\mathrm{halo}}(z)}{\rho_{\mathrm{c}}} = \frac{\rho_{\mathrm{M}}^{\mathrm{halo}}(z)\Omega_{\mathrm{m}}}{\rho_0}, \tag{4}$$

where $\Omega_{\mathrm{m}} = \rho_0/\rho_{\mathrm{c}}$ is the density parameter of the matter in the Universe and ρ_{c} is the critical density.

We can obtain a similar equation for the baryonic density parameter in the collapsed halos if we assume that the baryonic fraction in each halo is the same as the global value. Then Ω

contributed by baryons in collapsed halos within a logarithmic mass range [ln M, ln$(M+\mathrm{d}M)$] is (Choudhury and Padmanabhan 2002)

$$\Omega^{\mathrm{halo}}_{\mathrm{M,B}}(z) = \Omega^{\mathrm{halo}}_{\mathrm{M}}(z)\frac{\Omega_{\mathrm{B}}}{\Omega_{\mathrm{m}}} = -\left(\sqrt{\frac{\pi}{2}}\mathrm{e}^{-\nu^2/2}\right)\frac{\mathrm{d}\ln\sigma(M)}{\mathrm{d}\ln M}\nu(z, M)\,\mathrm{d}\Omega_{\mathrm{B}}. \tag{5}$$

Assuming that all the baryons are either converted into stars or gaseous cloud, we can write

$$\Omega_\star(z) = f_\star\Omega^{\mathrm{halo}}_{\mathrm{M,B}}(z). \tag{6}$$

The above equation is directly related to the density of mass in stars formed in the DM halos, namely

$$\Omega_\star(z) = \frac{\rho_\star(z)}{\rho_{\mathrm{c}}}, \tag{7}$$

with $f_\star$ being the efficiency of conversion of gas into stars; that is, it gives us the fraction of baryons present in the halos that are converted into stars.

3. The gravitational wave production from pre-galactic stars

The high-mass stars present in the DM halos will finish their lives as stellar black holes (probably Population III stellar black holes). This population will produce a stochastic background of gravitational waves whose dimensionless amplitude is given by (see de Araujo *et al* (2000, 2002) for details)

$$h^2_{\mathrm{BG}} = \frac{(7.4\times10^{-20}\alpha)^2\varepsilon_{\mathrm{GW}}}{\nu_{\mathrm{obs}}}\int_{z_{\mathrm{cf}}}^{z_{\mathrm{ci}}}\int_{m_{\mathrm{min}}}^{m_{\mathrm{u}}}\left(\frac{m}{M_\odot}\right)^2\left(\frac{d_{\mathrm{L}}}{1\,\mathrm{Mpc}}\right)^{-2}\dot{\rho}_\star(z)\frac{\mathrm{d}V}{\mathrm{d}z}\phi(m)\,\mathrm{d}m\,\mathrm{d}z. \tag{8}$$

The above equation takes into account the contribution of different masses that collapse to form black holes occurring between redshifts z_{ci} and z_{cf} (beginning and end of the star formation phase, respectively) that produce a signal at the same frequency ν_{obs}. We consider that the formation of these black holes occurs at redshift $5 \leqslant z \leqslant 30$. This choice is compatible with the present results of the WMAP satellite whose excess power is described by reionization at redshift $11 < z_{\mathrm{r}} < 30$ at 95 % confidence (Kogut *et al* 2003).

It is worth mentioning that equation (8) refers to the black hole 'ringing', which has to do with the de-excitation of the black hole quasi-normal modes. Note also that the efficiency of generation of GWs is $\varepsilon_{\mathrm{GW}} \propto a^4$ (see, e.g., Stark and Piran 1986), where 'a' is the dimensionless angular momentum. Thus, the greater the GW efficiency, the greater the dimensionless angular momentum. Other quantities present in (8) are: d_{L}, the luminosity distance to the source, $\mathrm{d}V$, the co-moving element volume, $\phi(m)$, the stellar initial mass function (IMF), and $\dot{\rho}_\star(z)$, the star formation rate (SFR) density.

In particular, we consider a Salpeter IMF, where the progenitor masses of the black holes are in the range $m = 25$–$125M_\odot$. The remnant and the progenitor masses are related to $m_{\mathrm{r}} = \alpha m$ and we assume $\alpha = 0.1$ (de Araujo *et al* 2000, 2002). Note that $\dot{\rho}_\star(z)$ is directly obtained from the Press–Schechter formalism. Thus, from the detection of a stochastic background, we can in principle resolve the inverse problem, namely, to obtain the star formation density. Then, we can make a comparison between the 'observational' $\dot{\rho}_\star(z)$ and that predicted by the Press–Schechter formalism. This comparison could be used for a better comprehension of the feedback process on the ambient medium and to know what the actual

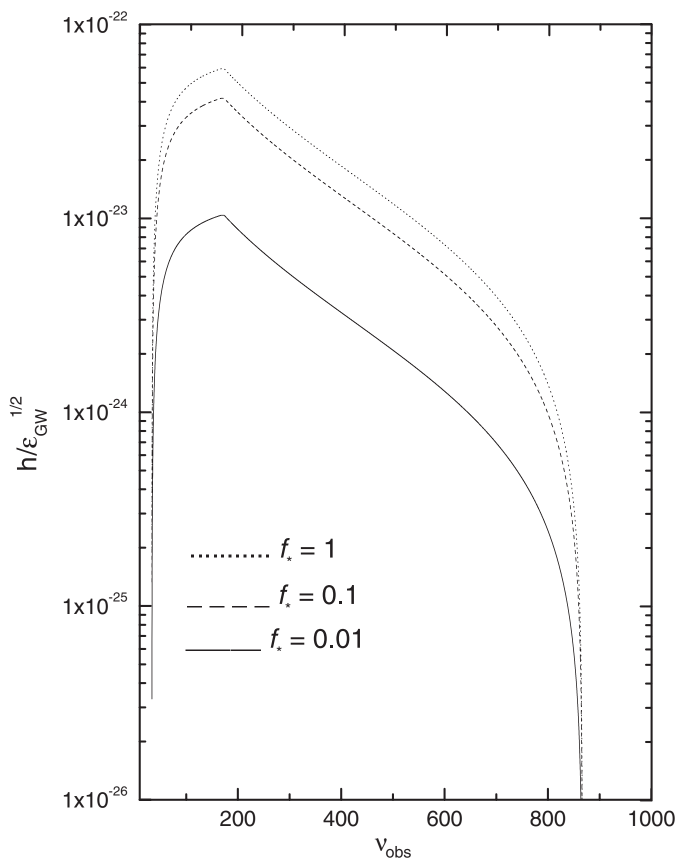

Figure 1. The stochastic background is present for different values of $f_\star$. Note that for $f_\star \leqslant 10^{-2}$ the S/N ratio is lower than unity even for a pair of LIGO III interferometers. Our results are presented for the following parameters: Hubble constant $h = 0.71$, baryonic density parameter $\Omega_B h^2 = 0.0224$, matter density parameter $\Omega_m h^2 = 0.135$, $\Omega_{total} = 1$ and a power spectrum with $n = 0.93$. Our model takes into account halos of mass $M > 10^8 M_\odot$.

IMF of the Population III stars is. In the following section we present the numerical results and discussions.

4. Numerical results and discussions

To calculate h_{BG} we adopted the standard Salpeter IMF. For ε_{GW}, the efficiency of production of GWs, whose distribution function is unknown, we have parametrized our results in terms of its maximum value, namely, $\varepsilon_{GW_{max}} = 7 \times 10^{-4}$. This figure is obtained from studies by Stark and Piran (1986) who simulated the axisymmetric collapse of a rotating star to a black hole.

In figure 1, we present the curve h_{BG} versus ν_{obs}. Knowing the frequency band $\nu_{min} - \nu_{max}$ detected from a cosmological source, we can obtain both z_{ci} and z_{cf}. Thus, these redshifts are therefore observable.

The background predicted in the present study cannot be detected by single forthcoming interferometric detectors, such as VIRGO and LIGO (even by advanced ones). However, it

Table 1. For different values of $f_\star$, the fraction of baryons converted into stars, we present the S/N for pairs of LIGO I, II and III ('first', 'enhanced' and 'advanced', respectively) observatories for one year of observation. (Note that an efficiency of generation $\varepsilon_{\mathrm{GW_{max}}} = 7 \times 10^{-4}$ is assumed.)

	S/N		
$f_\star$	LIGO I	LIGO II	LIGO III
1	2.8×10^{-2}	3.2	13
0.1	1.4×10^{-2}	1.6	6.3
0.01	8.5×10^{-4}	0.098	0.4

is possible to correlate the signal of two or more detectors to detect the background that we propose to exist (see a detailed discussion of this issue in de Araujo *et al* (2002)).

Note that for the 'initial' LIGO (LIGO I), there is no hope of detecting this background of GWs as can be observed in table 1. For the 'enhanced' LIGO (LIGO II) there is some possibility of detecting the background, since S/N > 1, if the fraction of baryons converted into stars is $0.1 < f_\star \leqslant 1$. Even if the LIGO II interferometers cannot detect such a background, it will be possible to constrain at least both the efficiency of conversion of baryons in stars and the efficiency of generation of gravitational waves $\varepsilon_{\mathrm{GW}}$.

The prospect of detection with the 'advanced' LIGO (LIGO III) interferometers is much more optimistic, since the S/N is significantly greater than unity for $f_\star > 0.1$. Only if the value of $f_\star$ were significantly lower than 0.1 would the detection not be possible.

5. Conclusions

We have shown that a background of GWs is produced from Population III black hole formation at high redshift. This background can in principle be detected by a pair of LIGO II (or more probably by a pair of LIGO III) interferometers.

The main idea of this work is to show that from the detection of a stochastic background, we can in principle obtain some insight on how star formation occurs at high redshifts.

One parameter that could be constrained is $f_\star(z)$, the fraction of baryons which are converted into stars. Thus, it will be possible to solve the inverse problem obtaining the shape of the star formation density. Then, we could make a comparison between the 'observational' $\dot{\rho}_\star(z)$ and that predicted by the Press–Schechter formalism. This comparison could be used for a better comprehension of the feedback process (cooling and heating) on the ambient medium and to know what the actual IMF of the Population III stars is.

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