Can lightning be a noise source for a spherical gravitational wave antenna?

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The detection of gravitational waves is a very active research field at the moment. In Brazil the gravitational wave detector is called Mario SCHENBERG. Because of its high sensitivity it is necessary to model mathematically all known noise sources so that digital filters can be developed that maximize the signal-to-noise ratio. One of the noise sources that must be considered are the disturbances caused by electromagnetic pulses due to lightnings close to the experiment. Such disturbances may influence the vibrations of the antenna's normal modes and mask possible gravitational wave signals. In this work we model the interaction between lightnings and SCHENBERG antenna and calculate the intensity of the noise due to a close lightning stroke in the detected signal. We find that the noise generated does not disturb the experiment significantly.

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I. SCHENBERG'S FEATURES

The detection of gravitational waves is presently an active research field and there are several detectors either being built or in operation around the world, involving countries like USA, Japan, Germany, The Netherlands, Italy and Australia. In Brazil the gravitational wave detector is called Mario SCHENBERG. This resonant-mass detector will operate in coincidence with some of the detectors in other countries. It is now in an advanced process of installation at the University of Sao Paulo, in the capital of Sao Paulo State. It is expected that SCHENBERG will be tested with three of its nine transducers in 2006.

Resonant-mass gravitational wave detectors like SCHENBERG detect the presence of a signal basically through the vibration of its first quadrupolar modes. Transducers strategically positioned on the antenna's surface are able to transform these vibrations into electric signals that can be digitally processed. As one can see from Fig. 1, SCHENBERG's antenna is spherical; it is made of a CuAl alloy (6% Al) and has 65 cm in diameter. More details on its functioning can be found in Ref. [1].

According to General Relativity the quadrupolar modes are expected to be excited by gravitational waves [2]. Because of its characteristics in SCHENBERG the first of these modes are tuned to ≈ 3.2 kHz. However, if an electromagnetic pulse generated by a lightning stroke hits the antenna most of its normal modes should be excited, including the quadrupolar ones. Therefore lightnings are expected to create noise in the detector perhaps masking detectable gravitational waves.

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In a previous work we have modeled the interaction between cosmic rays and SCHENBERG [3]. In that paper the interaction was basically thermodynamical. In this work the approach is different: the electromagnetic wave generated by the lightning stroke hits the antenna and transfers momentum to it. As a first approximation to the

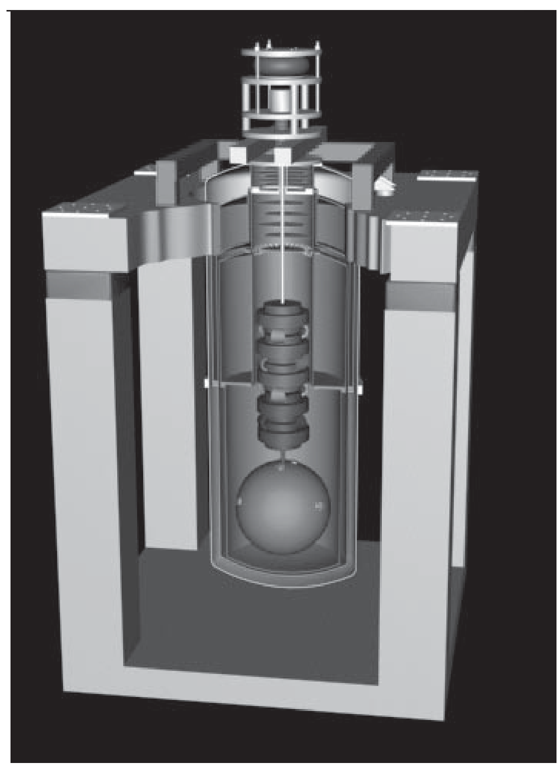

FIG. 1. Schematics of the SCHENBERG detector. The spherical antenna is carefully suspended inside metallic chambers that keep it in vacuum at 4 K.

calculations we will assume that SCHENBERG consists only of the spherical antenna, ignoring for now the cryogenic chambers.

In what follows, after a brief description of how SCHENBERG would respond to the influence of an electromagnetic wave we will present the mathematical model for lightning strokes. Then the lightning force density is calculated since it will be used in the model of the interaction between the detector and the lightning. Results and conclusions are the closing sections.

II. ELECTROMAGNETIC WAVES IN CONDUCTING MEDIA

A lightning creates an electromagnetic pulse that propagates with the speed of light. We want to investigate what happens when this pulse reaches the antenna, which is a conducting solid medium. For the description of the behavior of electromagnetic waves in conducting media one can use results found in the literature [4]. Two types of losses are encountered by an electromagnetic wave striking a metallic surface: (i) the wave is partially reflected from the surface (reflection loss), and (ii) the transmitted portion is attenuated as it passes through the medium. This latter effect is called absorption loss. Absorption loss is the same in either the near or the far field and for electric or magnetic fields. Reflection loss, on the other hand, is dependent on the type of field and the wave impedance.

The transition region between the near and the far fields is around $\lambda/2\pi$, where λ is the source's wavelength. In the case of SCHENBERG, the relevant wavelength is $\lambda = c/\nu_0 = 3 \times 10^8/3175 \simeq 9.4 \times 10^4$ m, thus implying a transition region around 15 km. We will consider distances smaller than 15 km because they include stronger strokes. Within this distance the far field limit would be suited only for higher frequencies, which are much weaker then the lower ones, as one can see from Fig. 2. For these reasons we conclude that the electromagnetic fields generated by lightnings that hit the ground close to the detector are in the near field limit.

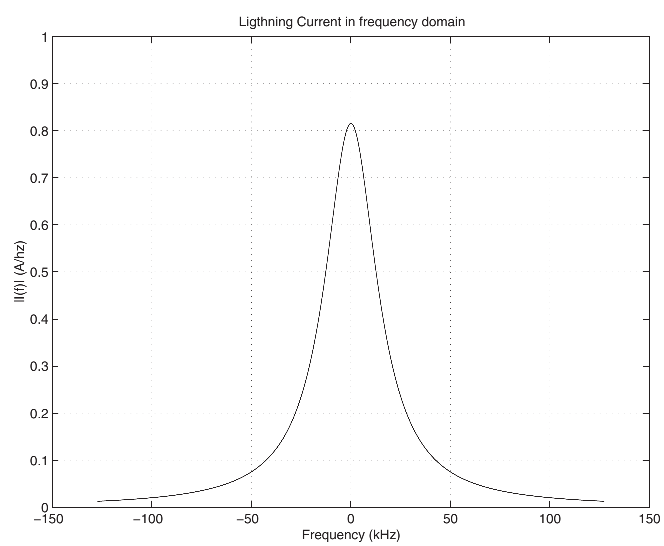

FIG. 2. Plot of the Fourier transform of the lightning's current.

A. Absorption loss

It can be shown that when an electromagnetic wave reaches a good conductor it is reduced to $1/e = 0.369$ of its initial amplitude after travelling the distance (all units in this paper are in the MKS system)

$$\delta \simeq \sqrt{\frac{2}{\mu\omega\sigma}}.$$

The distance δ is called *skin depth*. It varies with the frequency of the wave, as can be seen in Table I [5]. The symbols μ and σ refer, respectively, to the permeability (equal to $4\pi \times 10^{-7}$ H/m for free space) and to the conductivity (equal to 5.82×10^7 mhos/m for copper).

Then for a medium with thickness t the absorption loss is given by

$$A = 8.69(t/\delta) \quad dB.$$

In the case of SCHENBERG $\delta \sim 1.5$ mm. Thus an appreciable fraction of absorption loss should happen practically at the antenna's surface.

B. Reflection loss in the near field

In the near field the electric (E) and the magnetic (H) fields must be considered separately, since the ratio of the two is not constant. In this limit reflection loss varies with the wave impedance, defined by $Z \equiv E/H$. For the same source frequency a high-impedance (electric) field has a higher reflection loss than a low-impedance (magnetic) field.

In the case of SCHENBERG, mostly made of copper and sensitive to frequencies around 3.2 kHz, we can expect a minimum reflection loss around 50 dB [5]. In fact, for low frequency plane waves, as the ones considered here, reflection loss accounts for most of the attenuation.

TABLE I. Skin depth δ for copper and aluminum, in mm.

Frequency (Hz)	δ for Cu	δ for Al
60	8.51	10.90
100	6.60	8.46
1 k	2.08	2.67
3.2 k	1.63	1.49
10 k	0.66	0.84
100 k	0.20	0.28
1 M	0.08	0.08
10 M	0.02	0.02

III. MODEL FOR THE LIGHTNING

The phenomenon of lightning is still not fully understood but several models have already been proposed to explain it [6]. In brief, a cloud-to-ground lightning flash is usually composed of several intermittent discharges called strokes, which are made up of a leader phase and a return stroke phase. The leader initiates the return stroke, which is an upward travelling wave. A relatively large electric field exists in the leader-return stroke channel and this field produces ionization and results in a current. The power input renders the channel very luminous and causes its rapid expansion, producing thunder. More details on the phenomenon can be found in [7].

In our analysis we will consider the effect of a first return stroke on SCHENBERG. The idealized stroke is drawn in Fig. 3, adapted from [6].

The electric field intensity $\mathbf{E}$ and the magnetic flux intensity $\mathbf{B}$ produced by the channel current $i(z, t)$ at a distance D from the vertical channel of height H are given by [8]

$$\mathbf{E}(D, t) = \frac{1}{2\pi\epsilon_0}\left[\int_0^H \int_0^t \frac{2 - 3\sin^2\theta}{\rho^3} i(z, \tau - \rho/c) d\tau dz + \int_0^H \frac{2 - 3\sin^2\theta}{c\rho^2} i(z, t - \rho/c) dz - \int_0^H \frac{\sin\theta}{c^2\rho} \frac{\partial i(z, t - \rho/c)}{\partial t} dz\right]\mathbf{e}_z, \quad (1)$$

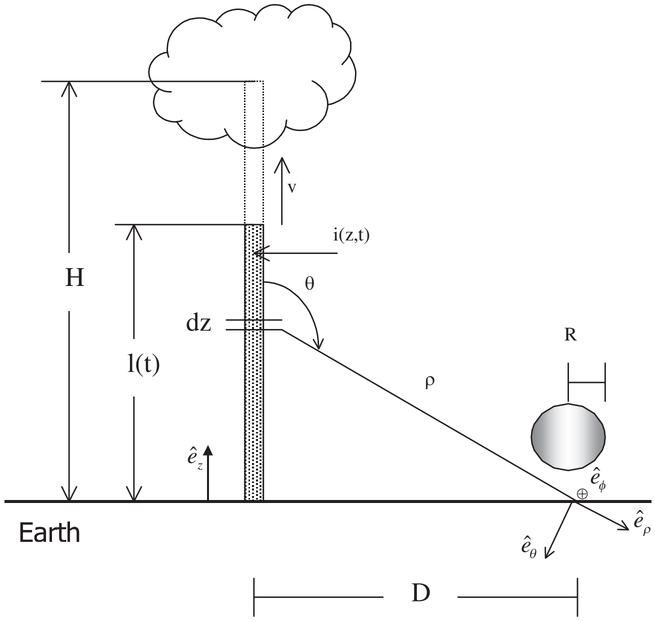

FIG. 3. Schematics of an idealized return stroke with the geometrical and physical parameters used in the calculations.

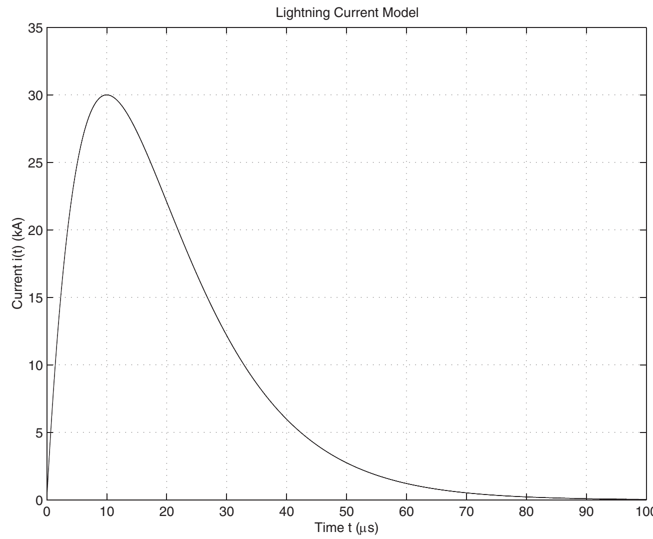

FIG. 4. Plot of the behavior of the current of the lightning as a function of time, modeled according to a Poisson distribution.

$$\mathbf{B}(D, t) = \frac{\mu_0}{2\pi}\left[\int_0^H \frac{\sin\theta}{\rho^2} i(z, t - \rho/c) dz - \int_0^H \frac{\sin\theta}{c\rho} \frac{\partial i(z, t - \rho/c)}{\partial t} dz\right]\mathbf{e}_\phi, \quad (2)$$

where ϵ_0 is the permittivity, μ_0 the permeability of free space and c is the speed of light in vacuum.

There are several models for the channel current $i(z, t)$ [6], all trying to describe actual strokes as accurately as possible. Since we are interested mostly in the intensity of the disturbance caused by a stroke in the detection of gravitational wave process we will use a simple expression for the current, which accounts for the basic features of a typical lightning.

We assume that the stroke has a typical length of 5 km. It takes 10μ s to reach the maximum current of 30 kA and lasts typically 50μ s. With these values we model the current as a function of time as a Poisson distribution:

$$i(t) = ate^{-\lambda t}, \quad (3)$$

with $a = 8.155 \times 10^9$ A/s and $\lambda = 10^5$ s^{-1}. See Fig. 4 for the plot of this function. We will adopt here the Bruce-Golde model [9], for which $i(z, t) = i(0, t)$. Also, we will assume that the lightning stroke is quite close to the detector, at a distance $D = 1500$ m, which ensures a high amplitude to the electromagnetic front that will disturb the antenna.

IV. DETERMINATION OF THE LIGHTNING'S FORCE DENSITY

From the expressions (1)–(3) we were able to compute the Poynting vector for the electromagnetic (e-m) radiation that reaches SCHENBERG using the formula $\mathbf{S}(t) = \mu_0^{-1}\mathbf{E}(t) \times \mathbf{B}(t)$. The coordinate system used is depicted

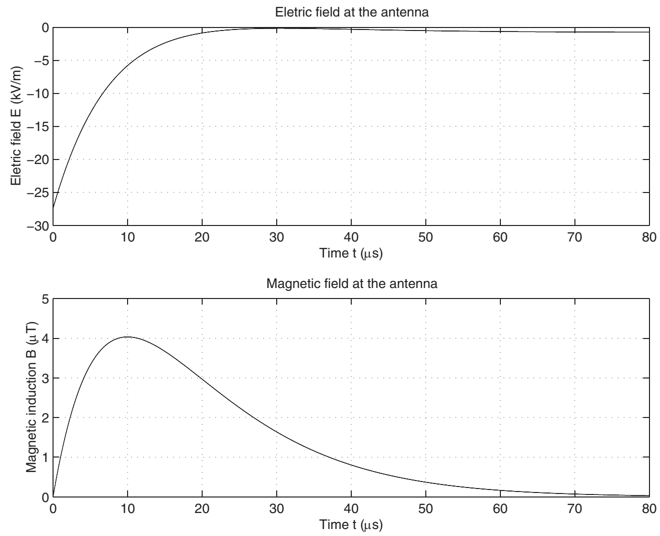

FIG. 5. Plots of the electric (a)and magnetic (b) fields as functions of time for D = 1500 m.

in Fig. 3, with D fixed at 1.5 km. From this expression we found the density of momentum of the wave, $\mathbf{p}(t) = \mathbf{S}(t)/c^2$. The plots of the electric and magnetic fields at this close distance are shown in Fig. 5.

By differentiating $\mathbf{p}(t)$ we obtained the expression for force density of the e-m wave, $\mathbf{f}(t)$, which for the distance we chose has modulus

$$f(t) = 2.21 \times 10^{-3} e^{-10^{-5}t}(t - 10^{-5}) + e^{2\times 10^{-5}t}(-2.83 \times 10^3 t^2 + 0.768t - 2.42 \times 10^{-7}) \quad (4)$$

with units of VTs^2m^{-3} in the MKSA system.

V. THE MODEL OF THE INTERACTION

The mechanical model of a spherical gravitational wave antenna has already been presented in the literature. We will follow here the notation on [10] for the case that there are no transducers coupled to the antenna and that only source of force on the sphere is the e-m wave. Then the Fourier transform of the radial amplitude of motion of the detector surface under the influence of this wave is given by

$$\tilde{a}_m(\omega) = kI_m(-\omega^2 + i\tfrac{\omega_0}{Q}\omega + \omega_0^2)^{-1}\tilde{f}(\omega), \quad (5)$$

where $\tilde{f}(\omega)$ is the Fourier transform of the force density, the constant k is defined by

$$k \equiv \alpha(R)R^2/(\varrho M_{\rm eff} N)$$

and

$$I_m = \int_{\theta=0}^{\pi}\int_{\phi=0}^{\pi} Y_{2m}(\theta, \phi)\sin^2\theta \sin\phi d\theta d\phi.$$

TABLE II. Values of the constants used in the calculations.

Name	Symbol	Value
Radius at 4 K	R	0.3239 m
Geometric constant	$\alpha(R)$	2.862
Density at 4 K	ϱ	8077.5 kg/m^3
Effective mass	$M_{\rm eff}$	288 kg
Normalization constant	N	0.142 kg^3
Quadrupole mode frequency	ν_0	3175 Hz
Mechanical quality factor	Q	2×10^7

The list of constants used in the equations is presented in Table II. They are based on recent data from SCHENBERG [11]. The functions $Y_{2m}(\theta, \phi)$ are the spherical harmonics and can be found in textbooks [4] . As usual, $\omega_0 = 2\pi\nu_0$. The limits of integration are such that they cover the half-sphere that is in the direction of the lightning stroke.

VI. RESULTS

From Eq. (5) we could determine the amplitude of the motion caused by the e-m wave ($a_m(t)$) in the five degenerate quadrupole modes of SCHENBERG, given by the index $m = -2, -1, 0, 1, 2$. We found that modes m = 1 and m = −1 are not disturbed at all by this wave. Modes m = 2 and m = −2 are equally disturbed, slightly more than mode m = 0.

In gravitational wave detection the adimensional amplitude $h_m = a_m/R$ is preferred as a measure of the antenna's motion. In terms of this figure the results we obtained on the motion of the modes with time is shown in Fig. 6.

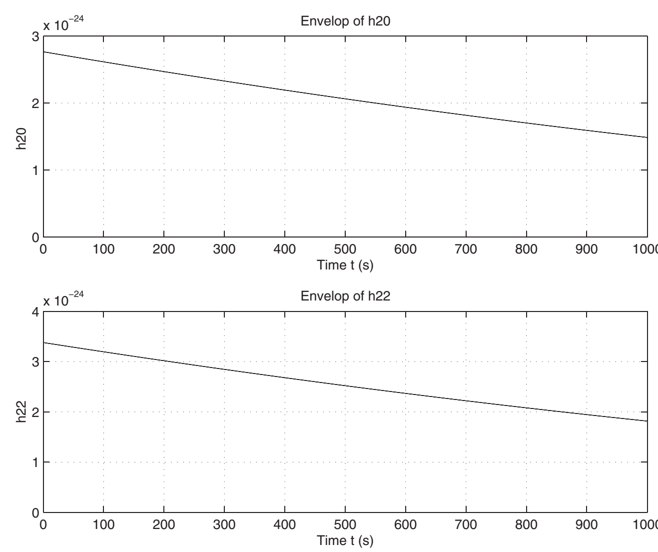

FIG. 6. Adimensional envelop of amplitudes as a function of time for modes (a) m = 0 and (b) m = 2. Notice that the amplitude decays slowly due to the antenna's high mechanical quality factor.

Notice that the modes present a maximum adimensional amplitude around 3.4×10^{-24}.

VII. CONCLUSIONS

The gravitational wave detector SCHENBERG is expected to reach a maximum sensitivity of $h \sim 10^{-20}$ for impulsive waves when fully operational in a few years. The noise caused by a typical, close lightning stroke on the detector was found in this work to be approximately 3 orders of magnitude smaller than this expected sensitivity.

Had the electromagnetic shielding due to the metallic cryogenic chambers be taken into account in the calculations this noise would be even less significant. Therefore we conclude that it is unlikely that lightning strokes should cause detectable noise while SCHENBERG is running.

This result is relieving since it may happen that SCHENBERG does not run in coincidence with other detectors from time to time, and it must be free of as much kinds of noise as possible. The region in which this detector is been built is prone to summer thunderstorms during approximately 2 months, accompanied by many lightning strokes. We have just shown that such strokes should not directly disturb the data significantly. Occasionally electric fluctuations may happen in power supply due to lighting strokes but these can be easily ruled out with the continuous monitoring of the experiment.

Although the maximum adimensional amplitude due to lighting strokes found above is larger than the amplitudes due to many monochromatic gravitational waves [2] this kind of noise should not disturb the detection of this kind of waves since the signal is integrated in time.

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