

## Astrophysics from data analysis of spherical gravitational wave detectors

César H. Lenzi · Nadja S. Magalhães ·  
César A. Costa · Rubens M. Marinho Jr ·  
Helmo A. B. Araújo · Odylio D. Aguiar

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**Abstract** The direct detection of gravitational waves will provide valuable astrophysical information about many celestial objects. Also, it will be an important test to general relativity and other theories of gravitation. The gravitational wave detector SCHENBERG has recently undergone its first test run. It is expected to have its first scientific run soon. In this work the data analysis system of this spherical, resonant mass detector is tested through the simulation of the detection of gravitational waves generated during the inspiralling phase of a binary system. It is shown from the simulated data that it is not necessary to have all six transducers operational in order to determine the source's direction and the wave's amplitudes.

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C. H. Lenzi (✉) · R. M. Marinho Jr · H. A. B. Araújo  
Departamento de Física, Instituto Tecnológico de Aeronáutica,  
Praça Marechal Eduardo Gomes 50, São José dos Campos, SP 12228-900, Brazil  
e-mail: chlenzi@ita.br

R. M. Marinho Jr  
e-mail: marinho@ita.br

H. A. B. Araújo  
e-mail: helmo@ita.br

N. S. Magalhães  
Centro Federal de Educação Tecnológica de São Paulo,  
R. Dr. Pedro Vicente 625, São Paulo, SP 01109-010, Brazil  
e-mail: nadjam@cefetsp.br

C. A. Costa · O. D. Aguiar  
Departamento de Astrofísica, Instituto Nacional de Pesquisas Espaciais,  
Av. dos Astronautas 1.758, São José dos Campos, SP 12227-010, Brazil  
e-mail: cesar@das.inpe.br

O. D. Aguiar  
e-mail: odylio@das.inpe.br

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## 1 Introduction

As predicted by the theory of relativity and other theories of gravitation, time-dependent gravitational forces are expected to propagate in spacetime in the form of waves [16]. Strong gravitational waves (g.w.) are expected to be generated by astrophysical objects [19]. For instance, two very massive stars orbiting each other would emit such waves. In particular, when they are coalescing they emit waves in a large frequency range. Another example is given by a black hole that rings down, also emitting in different wavelengths [5].

There is the belief within the international community that works with g.w. detection that the first direct detection of gravitation radiation from an astrophysical source will become a reality in the near future. This will open a new window to the universe, bringing new information about known objects and about fairly unknown things, like dark matter. In order to extract such information from the huge amount of data that is expected to be generated from the detectors' outputs (which has already started) a lot of work will be demanded in the field of data analysis.

From the analysis of the detected waves emitted by such sources important information is expected. The very first direct detection will provide a test for one of the predictions of the theory of general relativity. Then, continuous observation will allow testing other theories of gravitation [12, 14, 18], besides initiating gravitational astronomy [11, 17].

In order to make astrophysical observations the following parameters that characterize the g.w. are needed: the amplitudes of the two states of polarization of the wave as functions of time ( $h_+(t)$  and  $h_\times(t)$ ) and the source's direction in the sky (given by the angles  $\Theta$  and  $\Phi$ ) [8]. Therefore, gravitational wave observatories must be able to detect four independent observables in order to allow for gravitational wave astronomy to start.

The basis for the existence of resonant-mass gravitational wave detectors like SCHENBERG [2] is the fact that solid bodies are distorted in the presence of g.w. due to the changes in spacetime. In principle, the spherical geometry of SCHENBERG's antenna implies no preferred direction of observation (i.e., it is omnidirectional) and the all observables needed for gravitational astronomy can be obtained from only one spherical detector appropriately equipped [10, 11]. When fully operational, this detector will be able not only to acknowledge the presence of a g.w. within its bandwidth: it will be able to inform the direction of a source in sky, the wave's amplitude and the polarization components in the detector bandwidth—one only antenna working as a gravitational wave observatory in a bandwidth between 3,000 and 3,400 Hz, sensitive to displacements around  $10^{-20}$  m. To this end six transducers will continuously monitor radial displacements of the antenna's surface. SCHENBERG is installed at the Physics Institute of the University of São Paulo (São Paulo city, Brazil) and has undergone its first test run in September 8, 2006, with three transducers tuned to the above band.

There are mathematical models for this detector for the case that all these six transducers are operational. Such models have been investigated two situations: one in which the transducers are perfectly uncoupled [3, 10, 13] and another in which the transducers are somehow coupled to each other [15]. For both cases it is possible to solve the inverse problem and obtain the four astrophysical parameters needed if one assume General Relativity as the correct theory of gravitation.

This work presents the results of an investigation [9] carried on by the data analysis group within the GRAVITON project [1] (the one SCHENBERG is part of) aimed at developing a model of the detector with *less* than six transducers. This is an important issue, since occasionally not all six resonators may be operational simultaneously. In this case there is a break in the convenient symmetry among the transducers. Two approaches were under investigation: one considered the model already developed for 6 transducers [4] and simply reduced their number; the other, described here, considered the fewer transducers as independent devices and redesigned the mathematical model. It was found that the system sphere plus transducers is equivalent to a system of independent bar detectors with center-of-masses located at the sphere's center-of-mass ("cyclical" bars) [11]. It was possible to retrieve the relevant astrophysical information about a coalescing binary neutron star system both from the sphere coupled to six transducers and from a system considering the sphere's outputs as the outputs of transducers coupled to cyclical bar detectors (which will be called the "method of independent bars"). These results are detailed as follows.

## 2 Method of independent bars

It has already been shown theoretically by Magalhaes et al. [11] that any array of  $n$  cyclical bars will respond to a gravitational wave in a way similar as a non-noisy, high  $Q$  spherical antenna monitored by  $n$  transducers. In both cases, the projection  $R^L$  of a g.w. onto a detector can be expressed in terms of the internal product between two symmetric, traceless tensors [11]:

$$R^L = \sum_{i,j=1}^3 W_{ij} D_{ij}^L. \quad (1)$$

In this expression  $R^L$  are the outputs of the transducers ( $L = 1, 2, \dots, n$ , where  $n$  is the number of transducers positioned on the sphere's surface or, equivalently, the number of cyclical bar detectors under consideration). The tensor  $W$  contains only information concerning the gravitational wave, while  $D^L$  carries only information concerning the detector. For a *bar* detector, if  $\theta_L$  is the angle the detection point (where the  $L$ th transducer is located) makes with the local zenith and  $\phi_L$  is the azimuthal angle then one can show that

$$D^L = \begin{bmatrix} \sin^2 \theta_L \cos^2 \phi_L - \frac{1}{3} & \frac{1}{2} \sin^2 \theta_L \sin 2\phi_L & \frac{1}{2} \sin 2\theta_L \cos \phi_L \\ \frac{1}{2} \sin^2 \theta_L \sin 2\phi_L & \sin^2 \theta_L \sin^2 \phi_L - \frac{1}{3} & \frac{1}{2} \sin 2\theta_L \sin \phi_L \\ \frac{1}{2} \sin 2\theta_L \cos \phi_L & \frac{1}{2} \sin 2\theta_L \sin \phi_L & \cos^2 \theta_L - \frac{1}{3} \end{bmatrix}. \quad (2)$$

It is the knowledge of the wave’s tensor,  $W$ , that allows one to determine all astrophysical parameters,  $(h_+(t), h_\times(t))$  and  $(\Theta, \Phi)$ , as was shown in [11]. In order to determine  $W_{ij}$  one notices that, from general relativity, this tensor has six independent terms. It is then necessary to have at least six equations to determine these six unknowns. One equation is the traceless condition

$$W_{11} + W_{22} + W_{33} = 0, \tag{3}$$

the other five can be given by

$$D_{11}^L h_{11} + 2D_{12}^L h_{12} + 2D_{13}^L h_{13} + D_{22}^L h_{22} + 2D_{23}^L h_{23} + 2D_{33}^L h_{33} = 2R^L. \tag{4}$$

with  $L = 1, \dots, 5$ . As usual, at least five cyclical bars with one transducer each are needed. The six equations given by (3) and (4) can be written in matrix form as

$$\begin{bmatrix} D_{11}^1 & 2D_{12}^1 & 2D_{13}^1 & D_{22}^1 & 2D_{23}^1 & D_{33}^1 \\ D_{11}^2 & 2D_{12}^2 & 2D_{13}^2 & D_{22}^2 & 2D_{23}^2 & D_{33}^2 \\ D_{11}^3 & 2D_{12}^3 & 2D_{13}^3 & D_{22}^3 & 2D_{23}^3 & D_{33}^3 \\ D_{11}^4 & 2D_{12}^4 & 2D_{13}^4 & D_{22}^4 & 2D_{23}^4 & D_{33}^4 \\ D_{11}^5 & 2D_{12}^5 & 2D_{13}^5 & D_{22}^5 & 2D_{23}^5 & D_{33}^5 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} W_{11} \\ W_{12} \\ W_{13} \\ W_{22} \\ W_{23} \\ W_{33} \end{bmatrix} = \begin{bmatrix} R^1 \\ R^2 \\ R^3 \\ R^4 \\ R^5 \\ 0 \end{bmatrix} \tag{5}$$

or  $DW = R$ . As long as the matrix  $D$  is square and non-singular, from the experimental knowledge of  $R^L$  and  $D^L$  one can obtain the wave’s tensor  $W$  through the relation

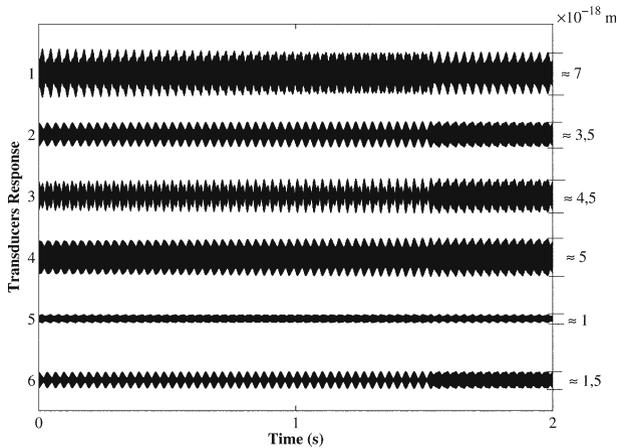
$$W = D^{-1}R. \tag{6}$$

In order to make sure that the use of a tensor for bars, (2), would yield the solution for the inverse problem using  $R^L$  derived from the data of a spherical detector, a test was performed. The transducers were positioned as planned in SCHENBERG, located according to the pentagonal faces of a truncated icosahedron:

$$[(\theta, \phi)] = [(79.1877^\circ, 120^\circ), (79.1877^\circ, 240^\circ), (79.1877^\circ, 0^\circ), (37.3774^\circ, 300^\circ), (37.3774^\circ, 180^\circ), (37.3774^\circ, 60^\circ)]. \tag{7}$$

For future use in the calculation of (6), in the construction of the matrix  $D$  only the transducers located in the five first positions were used.

The values of  $R^L$  were obtained using the software ‘‘Schenberg Simulator 2.0.0’’ [4], initially in the absence of noises. The mathematical model of the detector, which is the basis for the software, can be found, for instance, in [10]. In the software are used six two-mode transducers located on the antenna’s surface, distributed according to the angles listed above, operating simultaneously. This software has proven effective



**Fig. 1** Transducers' outputs from the simulation using as input a g.w. emitted by a coalescing NS–NS system

in solving the inverse problem [4]. The outputs of five of these transducers (the same used to construct the matrix  $D$ ) will be used as the components of  $R^L$ .

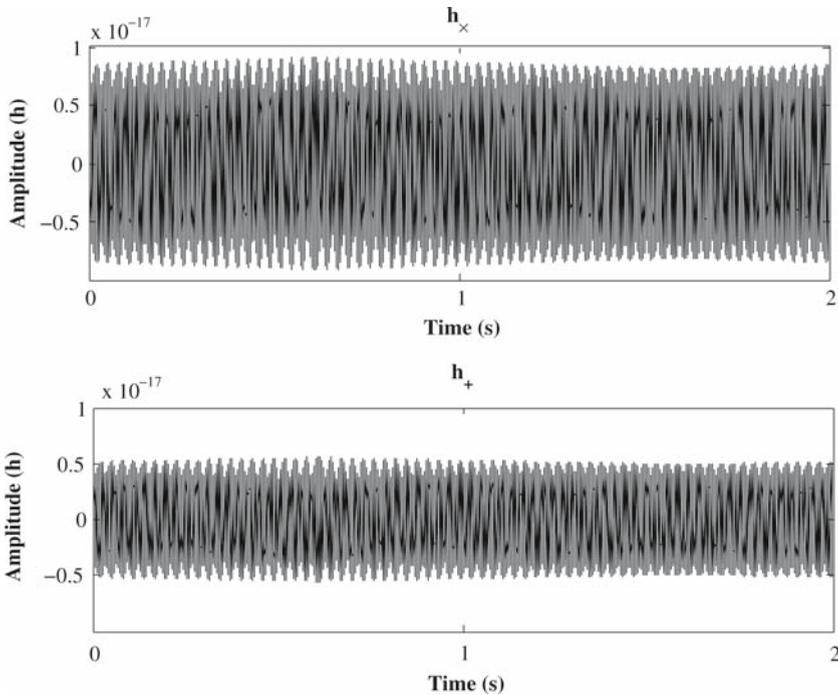
If the equivalence between the spherical detectors and the independent (cyclical) bars is correct, then the solution of the inverse problem with the bars should yield the astrophysical parameters of a source introduced as the input of the software that simulates SCHENBERG's response. As will be shown, this equivalence does exist.

The simulation that is going to be analysed here was run with the source represented by a waveform resultant from a coalescing neutron star (NS) binary system [6,7], incident on the detector according to the angles  $\Theta = 60^\circ$  and  $\Phi = 45^\circ$  relative to the same system that defines the transducers' positions. With the input from this source the software yielded the outputs illustrated in Fig. 1.

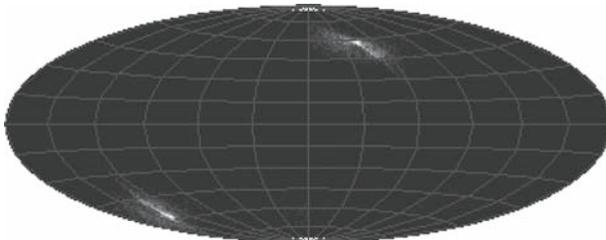
Using the first all the six outputs in Eq. (6) the values of  $W$  were found and, consequently [11], the source's direction was retrieved. In this particular case considered we have a probability of 68.27% that the direction is  $\Theta = 60^\circ \pm 2^\circ$  and  $\Phi = 44.4^\circ \pm 1.4^\circ$ . For the case of five outputs in Eq. (6) the result of the source's direction is  $\Theta = 60^\circ \pm 2^\circ$  and  $\Phi = 44.2^\circ \pm 1.6^\circ$ . It was verified that the result was in perfect agreement with the source's input angles. The wave's amplitudes were also determined, as shown in Fig. 2.

Different combinations of five transducers were used, yielding the same results as expected [10]. It became clear that the method of independent bars using five outputs was as effective in solving the inverse problem as the conventional method, which considers all six transducers simultaneously.

The procedure was repeated in the presence of Gaussian noises and the results from both approaches coincided again. See Fig. 3 for the case when the signal-to-noise ratio, SNR, is approximately 18. In this figure two regions of the sky are marked because there is a natural ambiguity in the determination of the direction by only one detector; if more than one detector is used then that there is a time delay between detections



**Fig. 2** Amplitudes of the g.w. emitted by the simulated coalescing NS–NS system. These plots were obtained from the solution of the inverse problem with the method of independent bars using five outputs in the absence of noise



**Fig. 3** Source's direction as determined by the method of independent bars using five bars in the presence of Gaussian noise with  $\text{SNR} \approx 18$ . The different degrees of gray are related to the differences in the energy absorbed by the sphere

and one can determine whether the wave came from up or down. The wider the region marked, the higher the SNR.

The method of independent bars was also tested using the output of only four transducers to solve the inverse problem in the presence of Gaussian noise for the case when SNR is the approximately 93. The preliminary results show that even using less transducers all the four astrophysical parameters were determined from the method. The result of the source's direction in this case is  $\Theta = 57^\circ \pm 5^\circ$  and  $\Phi = 42^\circ \pm 6^\circ$ .

This result was obtained using the following transducers:

$$[(\theta, \phi)] = [(79.1877^\circ, 120^\circ), (79.1877^\circ, 240^\circ), (79.1877^\circ, 0^\circ), (37.3774^\circ, 60^\circ)]. \quad (8)$$

However, in this case the computational time increases considerably. Also, the precision of the results varies with the choice of the transducers, confirming previous investigation [10].

### 3 Summary

Presently, two spherical detectors exist (SCHENBERG and MiniGrail), and they are expected to begin scientific runs soon. This work presents results of a method that allows determination of astrophysical parameters from the data of a spherical g.w. detector using six, five or four transducers. This is an original method, since in the literature only six transducers were considered so far.

The research will continue with the solution of the inverse problem using the method of independent bars with the output of four and three transducers, both assuming a noiseless and a noisy detector. Circular and elliptical polarizations should be considered, in addition to the linear ones. The detailed presentation of these results will be published in a longer paper that is now in preparation.

Also, it is intended to fully investigate SCHENBERG's directivity pattern, including the simulation of real data from signals arriving from different directions in the sky. This will allow statistical estimates of the actual detector performance based on the theoretical principles exposed here. Such an statistical analysis is essential since the less transducers one has, the more important, if not crucial, is to assess the significance of the detection.

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