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A noise model for the Brazilian gravitational wave detector 'Mario Schenberg'

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Abstract

'Mario Schenberg' is a spherical resonant-mass gravitational wave (GW) detector that will be part of a GW detection array of three detectors. The other two will be built in Italy and in The Netherlands. Their resonant frequencies will be around 3.2 kHz with a bandwidth of about 200 Hz. This range of frequencies is new in a field where the typical frequencies lay below 1 kHz, making the development of the mechanical system much more complex. In this work, a noise model of the detector is presented, where all main sources of noise were taken into account. The final goal is to calculate the expected sensitivity of the detector, analysing which parameters must be changed to improve this.

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1. Introduction

'Mario Schenberg' is a spherical resonant-mass gravitational wave detector cooled down to 20 mK, weighing 1150 kg, being built in the Department of Physics of Materials and Mechanics of the University of Sao Paulo and is expected to start operating in 2004. The sphere, 65 cm in diameter, is made of a copper–aluminium alloy [1] with 94% Cu and 6% Al. The distribution of motion sensors on the surface of the sphere will be based on the work by Merkowitz and Johnson [2]. Motion sensors are devices that monitor the motion of the sphere surface. The detector will have six motion sensors, also called transducers, arranged

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on the sphere surface in a half-dodecahedron distribution; the sensors will be located as if at the centre of the six connected pentagons in a dodecahedron surface. By analysing the signal of these sensors the intensity and the direction of the incoming gravitation wave can be obtained [3].

Two other similar detectors have been built or are planned: one in Italy called 'Sfera' and the other in The Netherlands called 'MiniGrail' [4]. Together these three detectors will form an array of GW detectors sensitive to frequencies around 3.2 kHz with a bandwidth of about 200 Hz.

The Brazilian group has decided to use as motion sensors microwave parametric transducers, similar to the ones used in the GW resonant-mass detector NIOBE by the Australian GW group [5]. For each of the six transducers, a resonator with two mechanical modes will be used as an impedance match between the sphere surface and a microwave cavity. The first of these modes, the one connected to the sphere, will be called the intermediate mode and the second one will be called the final mode, all made in a monolithic structure.

2. The transducer model

The microwave parametric transducer can be described as a microwave resonant cavity pumped with a 10 GHz microwave source. One of the sides of the cavity is part of the final transducer mode [6]; as the size of the cavity changes so does its resonant frequency, thus changing the amplitude of the sidebands in the output signal of the cavity. This change is proportional to the signal in the detector.

This kind of transducer was extensively studied by Tobar and Blair [7]. The noise in this kind of transducer can be divided into two groups: narrow band and broad band noises. The narrow band noise includes the back action and the Brownian noises. The broad band noise includes two different kinds of noise that will be detailed below. In the following expressions we will assume the situation in which the transducer electrical coupling coefficient is equal to one.

The back action noise is due to noise in the amplitude of the pump signal (S_{am}), and can be calculated using the incident signal pump power in the cavity (P_{inc}), the electrical quality factor of the cavity (Q_{e}), the frequency tuning coefficient ($\mathrm{d}f/\mathrm{d}x$) and the frequency of the pump (F_{pump}), giving the following expression for the back action noise spectral density:

$$S_{\mathrm{backaction}} \sim \frac{P_{\mathrm{inc}}^2}{4\pi^2}\left(\frac{Q_e\,\mathrm{d}f/\mathrm{d}x}{F_{\mathrm{pump}}^2}\right)^2 S_{\mathrm{am}}. \tag{1}$$

This expression represents the spectral density of the back action force applied to the test mass, the motion of which is monitored by the transducer, and is valid only if the frequency of the oscillator is equal to the resonant frequency of the transducer, which is not the best choice as it may result in a parametric instability of the antenna. If the pump oscillator frequency is not equal to the transducer resonant frequency, phase fluctuations of the pump oscillator will also be contributing to the back action force acting on the antenna.

The broad band noise can be divided into two components, one due to the amplifier that depends also on the noise temperature of the amplifier (T_{amp}) and can be calculated using the following expression:

$$S_{\mathrm{SerialAmp}} \sim \frac{k_b T_{\mathrm{amp}}}{P_{\mathrm{inc}}}\left(\frac{F_{\mathrm{pump}}}{Q_e\,\mathrm{d}f/\mathrm{d}x}\right)^2. \tag{2}$$

This equation represents the spectral density of the equivalent displacement fluctuations of the test mass related to the finite effective noise temperature of the microwave amplifier.

The second component of the broad band noise is due to phase noise in the pump microwave souce, since all signals in the sidebands are due to changes in phase because of the motion on one side of the cavity. The pump phase noise spectral density can be calculated using the phase noise spectral density in the pump (S_{pm}) and the detection frequency (f), according to the following expression:

$$S_{\text{phasenoise}} \sim S_{\text{pm}} \frac{f^2}{(\mathrm{d}f/\mathrm{d}x)^2}. \tag{3}$$

This equation describes the spectral density of the equivalent displacement fluctuations of the test mass related to pump oscillator phase noise.

A multimode transducer would have several advantages over the single mode transducer, as has been well discussed several times [8–10]. These advantages include increasing the detector's bandwidth and improving the electro-mechanical coupling. In order to improve these characteristics it is necessary to keep the mass ratio between the sphere effective mass [2] and the effective mass of the intermediate resonator mode equal to the ratio between the mass of the intermediate and the mass of the final resonator modes.

3. The detector model

The mechanical model used in this work for the spherical detector is the Merkowitz and Johnson [2] one. This model can be represented in a matrix format as the following

$$\begin{pmatrix} M_s\mathbf{I} & \mathbf{0} & \mathbf{0} \\ M_{r1}\alpha\mathbf{B}^T & M_{r1}\mathbf{I} & \mathbf{0} \\ M_{r2}\alpha\mathbf{B}^T & M_{r2}\mathbf{I} & M_{r2}\mathbf{I} \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{a}} \\ \ddot{\mathbf{q}}_1 \\ \ddot{\mathbf{q}}_2 \end{pmatrix} + \begin{pmatrix} k_s\mathbf{I} & -k_{r1}\alpha\mathbf{B} & \mathbf{0} \\ \mathbf{0} & k_{r1}\mathbf{I} & -k_{r2}\mathbf{I} \\ \mathbf{0} & \mathbf{0} & k_{r2}\mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{q}_1 \\ \mathbf{q}_2 \end{pmatrix}$$
$$= \begin{pmatrix} \mathbf{I} & -\alpha\mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & -\mathbf{I} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{F}^{GW} + \mathbf{F}_e^N \\ \mathbf{F}_1^N \\ \mathbf{F}_2^N \end{pmatrix}$$

where the first matrix has the mass terms, the second matrix has the amplitudes of acceleration of the sphere quadrupole modes and of the transducer modes, the third matrix has the effective spring constants for the model, the fourth matrix has the amplitudes of motion for all modes, and the last two have the force terms.

Using the procedures presented in [11] with a lump element model, we calculated for different mass ratios the sensitivity of the detector, and we chose the mass ratio that implies the best sensitivity and the transducer having a good size to be machined.

4. Predicted sensitivity

Using the procedures above and incorporating expressions for the Brownian, back action, amplifier and phase noises used for the microwave parametric transducer (equations (1)–(3)), a noise spectral density curve can be plotted together with the contributions of the different sources of noise, as can be seen in figure 1. The present goal parameters for the microwave parametric transducer are: electrical quality factor of 1000, microwave pump frequency of 10.21 GHz, frequency tuning coefficient ($\mathrm{d}f/\mathrm{d}x$) of 6×10^{14} Hz m^{-1}, amplifier temperature of 8 K, microwave phase noise of -131 dBc Hz^{-1} and microwave amplitude noise of -140 dBc Hz^{-1}. In this calculation the mass of the intermediate transducer mode was

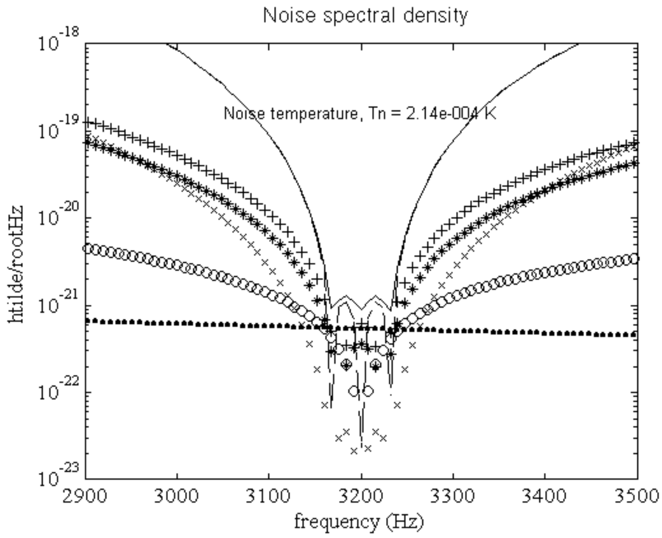

Figure 1. The strain spectral density curve for the detector at 4.2 K where the total noise is represented by the solid line, the partial noises are represented by the following lines: the sphere Brownian noise is the dotted line, the Brownian noise of the intermediate transducer mode is the 'o' line, the Brownian noise of the final transducer mode is the '+' line, the microwave back-action is the '$*$' line, the amplifier noise is the dashed line and the microwave phase noise is the '$\times$' line. Also, the expected noise temperature is shown.

53 g and the mass of the final transducer mode was 0.01 g. In this first calculation the thermodynamical temperature of the detector is 4.2 K and the mechanical quality factor for all modes is around 1 million (1.4 million for the sphere). These are the goal parameters for the first complete run of the detector. In this case the dominant noise is the one due to the amplifier but it is very close to the limit set by the thermal noise in the sphere.

For the future, to improve the sensitivity some parameters will change. As an example of the sensitivity in the future, the following parameters were chosen to be improved: the electrical quality factor, the microwave phase noise, the microwave amplitude noise, the amplifier temperature and the thermodynamical temperature of the detector. The chosen values are 500 000, -150 dBc Hz^{-1}, -160 dBc Hz^{-1}, 4.2 K and 15 mK, respectively. The noise spectral density for these conditions is shown in figure 2. In this case the dominant noise is also the amplifier noise very close to the sphere thermal noise.

5. Comments and future work

The Schenberg detector with the parameters presented above for the first complete run is expected to attain a noise temperature close to 1 μK. As can be seen in figure 1, the sources of noise present almost the same values in this case inside the detection band (around 3200 Hz). The only ways to improve the sensitivity in this case are either to increase the bandwidth of the detector or to decrease its thermodynamical temperature. The bandwidth cannot be increased much, but the temperature can be decreased to the order of tens of mK. By decreasing the temperature there is room to improve other parameters, and even to reach the standard quantum limit, if the noise temperature goes to $\cong$0.15 μK. In future works it is worth investigating the influence of different values for the transducer electrical coupling coefficient on the results.

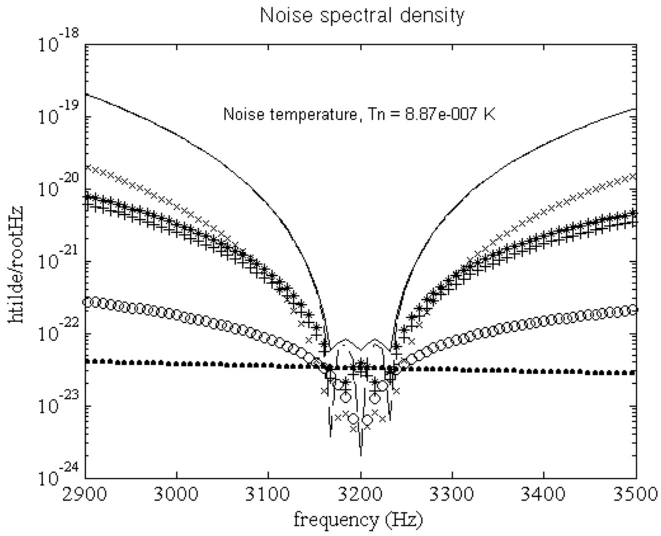

Figure 2. The strain spectral density curve for the simulated situation of the detector at 15 mK. For the key to the symbols see the caption to figure 1.

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