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Detectability of *f*-mode unstable neutron stars by the Schenberg spherical antenna

José Carlos N de Araujo[1], **Oswaldo D Miranda**[1,2] **and Odylio D Aguiar**[1]

[1] Instituto Nacional de Pesquisas Espaciais—Divisão de Astrofísica, Av. dos Astronautas 1758, São José dos Campos, 12227-010 SP, Brazil
[2] Instituto Tecnológico de Aeronáutica—Departamento de Física, Praça Marechal Eduardo Gomes 50, São José dos Campos, 12228-900 SP, Brazil

E-mail: jcarlos@das.inpe.br, oswaldo@das.inpe.br and odylio@das.inpe.br

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Abstract

The Brazilian spherical antenna (Schenberg) is planned to detect high frequency gravitational waves (GWs) ranging from 3.0 kHz to 3.4 kHz. There is a host of astrophysical sources capable of being detected by the Brazilian antenna, namely: core collapse in supernova events; (proto)neutron stars undergoing hydrodynamical instability; *f*-mode unstable neutron stars, caused by quakes and oscillations; excitation of the first quadrupole normal mode of 4–9 solar mass black holes; coalescence of neutron stars and/or black holes; exotic sources such as bosonic or strange matter stars rotating at 1.6 kHz; and inspiralling of mini black-hole binaries. We here address our study in particular to neutron stars, which could well become *f*-mode unstable producing therefore GWs. We estimate, for this particular source of GWs, the event rates that in principle can be detected by Schenberg and by the Dutch Mini-Grail antenna.

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1. Introduction

The Brazilian antenna is made of Cu–Al (94–6%), it has a diameter of 65 cm and covers the 3.0–3.4 kHz bandwidth using a two-mode parametric transducer (see Aguiar *et al* 2002, 2004 for details).

The initial target is a sensitivity of $\tilde{h} \sim 2 \times 10^{-21}$ Hz$^{-1/2}$ in a 50 Hz bandwidth, which we believe is reachable at 4.2 K with conservative parameters. An advanced sensitivity will be pursued later, cooling the antenna to 15 mK and improving all parameters (see Frajuca *et al* 2004).

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The Brazilian antenna will operate in conjunction with the Dutch Mini-Grail antenna, and the laser interferometer detectors, which can also cover such high frequencies with similar sensitivities.

It is worth mentioning that the event rates for the two spherical antennas will be the same, as long as they have the same sensitivity. So, throughout the paper, when we refer to the sources and the event rates for the Schenberg antenna, the reader has to bear in mind that we are also referring to the Dutch Mini-Grail antenna.

In a previous work by an MSc student of our group (Castro 2003), a preliminary study was conducted of the most probable sources of gravitational waves (GWs) that could be detected by Schenberg, namely: core collapse in supernova events; (proto)neutron stars undergoing hydrodynamical instability; f-mode unstable neutron stars, caused by quakes and oscillations; excitation of the first quadrupole normal mode of 4–$9M_{\odot}$ black holes; coalescence of neutron stars and/or black holes; exotic sources such as bosonic or strange matter stars rotating at 1.6 kHz; and inspiralling of mini black-hole binaries.

Our aim here is to present the results of this study and to revisit the f-mode unstable neutron star source of GWs. This specific pulsation mode is one of the most important channels for the emission of GWs (Kokkotas and Andersson 2001) by neutron stars. In particular, we are using recent results for the radial distribution of pulsars in the Galaxy (Yusifov and Küçük 2004) in order to determine the event rate detectable by Schenberg for a given efficiency of generation of GWs.

The paper is organized as follows. In section 2 we briefly consider the sources detectable by the Schenberg antenna, in section 3 we revisit the f-mode neutron star GW detectability by the Schenberg antenna, and finally in section 4 we present our conclusions.

2. The sources for Schenberg

First of all, it is worth mentioning that in the present estimates of event rates we are assuming that the Schenberg's sensitivity to burst sources is $h \sim 10^{-20}$, which seems reasonable from our projected $\tilde{h}$ and bandwidth.

It is important to bear in mind that such a sensitivity is not the quantum limit one, which could be a factor around 5 better. Also, it is worth remembering that using the 'squeezing technique' the quantum limit could in principle be overtaken. All this would imply that Schenberg, which will be using parametric transducers, could in principle present event rates significantly greater than those presented here; in some cases, the rates could well be higher by a factor of up to $\sim 10^2$.

It is worth stressing that the Brazilian detector will be sensitive to sources of the local group of galaxies ($r \sim 1.5$ Mpc). Although, just the Galaxy, M31 and M33 can give a significant contribution to the event rates, because these three galaxies account for more than 90% of the mass of the local group.

Our estimates show that, except for the mini black-hole binaries and the f-mode unstable neutron stars, the other putative sources for Schenberg present event rates of at most one event every $\sim$10 yr, at a signal-to-noise ratio (SNR) equal to unity. Thus, the prospect for the detection of these sources is not very promising. Because of this we do not enter into details of such estimates.

For the mini black-hole binaries we refer the reader to the paper by de Araujo *et al* (2004) for details. They show that the event rate in this case may be one event every 5 yr, at a SNR equal to unity.

Our main aim in this paper is to pay attention to the f-mode unstable neutron star, which can in principle be an important source for the Schenberg antenna. In our previous study we showed that one such event every year would be detectable by Schenberg at a SNR equal to unity.

In the next section we revisit the f-mode unstable neutron star study concerning its detectability in particular by the Schenberg antenna.

3. GWs from *f*-mode unstable neutron stars

Before studying the f-mode unstable neutron star as a source of GWs for the Brazilian antenna, which is of major interest here, it is worth considering its relevance as compared to the other pulsation modes such stars could have as regards the generation of GWs.

Relativistic stars are known to have a host of pulsation modes. Only a few of them, however, are of relevance for GW detection. From the GW point of view the most important modes are the fundamental (f) mode of fluid oscillation, the first few pressure (p) modes and the first GW (w) mode (Kokkotas and Schutz 1992). Among these three modes the pulsation energy is mostly stored in the f-mode in which the fluid parameters undergo the largest changes. It is worth mentioning that the r-mode can also be, under certain circumstances, a very important source of GWs (Andersson *et al* 2001).

An important question is how the modes are excited in the neutron stars, which is our concern here. There are many scenarios that could lead to significant asymmetries. A supernova explosion is expected to form a wildly pulsating neutron star that emits GWs. A pessimistic estimate for the energy radiated as GWs indicates a total release equivalent to $<10^{-6}M_{\odot}$. An optimistic estimate, where the neutron star is formed, for example, from strongly non-spherical collapse, suggests a release equivalent to $10^{-2}M_{\odot}$.

Another possible excitation mechanism for neutron star pulsation is a starquake, which can be associated with a pulsar glitch. The energy released in this process may be of the order of the maximum mechanical energy stored in the crust of the neutron stars, which is estimated to be 10^{-9}–$10^{-7}M_{\odot}$ (Blaes *et al* 1989, Mock and Joss 1998).

During the coalescence of two neutron stars several oscillation modes could in principle be generated. Stellar oscillations being excited by the tidal fields of the two stars, for example.

The neutron star may undergo a phase transition leading to a mini-collapse, which could lead to a sudden contraction during which part of the gravitational binding energy of the star would be released, and, as a result, it could occur that part of this energy would be channelled into pulsations of the remnant. Similarly, the transformation of a neutron star into a strange star is likely to induce pulsations.

In our previous study we have found that Schenberg could in principle detect at least one such source per year at a SNR equal to unity. The basic assumptions in this study are the following.

Firstly, we have taken into account in our estimate only the known pulsars. Secondly, we have assumed that the energy release in GWs is of the order of $10^{-6}M_{\odot}$ (see, e.g., Andersson and Kokkotas 1996). Thirdly, we have associated the f-mode excitation with the same mechanism responsible for the glitch phenomenon, which is related to some neutron star internal structure rearrangement (see, e.g., Horvath 2004).

Last but not least, we have assumed that the f-mode instability may produce GWs in the frequency band of 3.0–3.4 kHz, that of Schenberg. We refer the reader to the paper by Kokkotas and Andersson (2001), in particular its figure 2, where it is shown clearly that for a family of equations of state (EOSs) GWs of ∼3 kHz may be produced by f-mode unstable neutron stars.

It is worth recalling that GWs produced in the f-mode excitation depend on the EOS for the neutron star matter that, as is well known, is not completely established.

Before considering the improvements we intend to take into account in revisiting this study, it is worth recalling that the characteristic amplitude of GWs related to the f-mode instability is given by

$$h \simeq 2.2 \times 10^{-21} \left(\frac{\varepsilon_{\mathrm{GW}}}{10^{-6}}\right)^{1/2} \left(\frac{2\,\mathrm{kHz}}{f_{\mathrm{GW}}}\right)^{1/2} \left(\frac{50\,\mathrm{kpc}}{r}\right), \tag{1}$$

(see, e.g., Andersson and Kokkotas 1998) where $\varepsilon_{\mathrm{GW}}$ is the efficiency of generation of GWs, f_{GW} is the GW frequency and r is the distance to the source.

It is worth mentioning that the f-mode is a burst source of GWs with a duration of tenths of a second (see, e.g., Andersson and Kokkotas 1998). This signal being concentrated in a bandwidth which is completely within Schenberg's bandwidth. As a result, in the calculation of the SNR it is a good approximation to compare directly the detector pulse sensitivity with the predicted wave amplitude of the f-mode GW.

The Schenberg's sensitivity for burst sources can be of the order of 10^{-20}. This implies that, for $\varepsilon_{\mathrm{GW}} \sim 10^{-6}$ and $f_{\mathrm{GW}} = 3$ kHz, Schenberg can in principle detect f-mode unstable neutron star sources at distances of up to $r \sim 10$ kpc at SNR ~ 1.

Certainly, the number of neutron stars within the volume *seen* by Schenberg could be in principle enormous. Unless the efficiency of the generation of GWs through f-mode instability is $\ll 10^{-6}$ or such a mode is not excited at all, Schenberg could in principle detect f-mode unstable neutron stars with a considerable event rate.

In this study the main ingredient we have taken into account is the distribution function of pulsars in the Galaxy. One could consider, however, that it would be desirable to take into account, instead, a distribution function for the neutron stars in the Galaxy, because the pulsar population is a tiny part (say 0.1–0.01%; later on we explain how these figures are obtained) of the neutron star population. But, one has to bear in mind that most of the observed neutron stars are in fact seen in the form of pulsars.

It is worth mentioning that if the f-mode instability occurs in any neutron star, it being a pulsar or not, the event rate seen by any GW detector sensitive to its frequency could be strongly enhanced.

The distribution function for the pulsars in the Galaxy has been studied in many papers (see, e.g., Narayan 1987, Paczynski 1990, Hartman *et al* 1997, Lyne *et al* 1998, Schwarz and Seidel 2002, among others).

We here adopt the distribution function recently obtained by Yusifov and Küçük (2004), namely,

$$\rho(R) = A \left(\frac{R}{R_{\odot}}\right)^{a} \exp\left[-b\left(\frac{R - R_{\odot}}{R_{\odot}}\right)\right], \tag{2}$$

where $\rho(R)$ is the surface density of pulsars, R is Galactocentric distance, $R_{\odot} = 8.5$ kpc is the Sun–Galactic centre (GC) distance. Note that equation (2) implies that $\rho(0) = 0$, which is inconsistent with observations. To avoid such a problem the authors include an additional parameter R_1 and used a shifted gamma function, replacing R and $R_{\odot}$ in equation (2) by $X = R + R_1$ and $X_{\odot} = R_{\odot} + R_1$, respectively. The best fit, using the LMS method, gives $A = 37.6 \pm 1.9$ kpc^{-2}, $a = 1.64 \pm 0.11$, $b = 4.01 \pm 0.24$ and $R_1 = 0.55 \pm 0.10$ kpc. We refer the reader to the paper by Yusifov and Küçük (2004) for further details.

In figure 1 we present the number of pulsars as a function of the distance from the Sun, which has been obtained through integration of equation (2). Also present is the number of pulsars corrected by the beaming factor, which multiples that number by a factor of approximately 10 (see, e.g., Tauris and Manchester 1998).

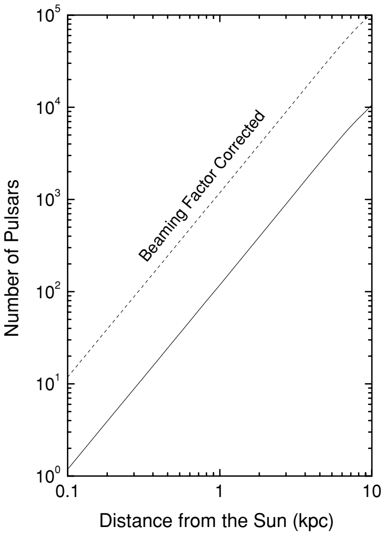

Figure 1. Number of pulsars as a function of the distance from the Sun without (solid line) and with (dashed line) the beaming factor correction.

The number of pulsars that could be seen by Schenberg $\sim 10^5$ (taking into account the beaming correction). In the whole Galaxy the number of pulsars with luminosity greater than 0.1 mJ y kpc^2 at 1400 MHz is predicted to be $\sim 2.4 \times 10^5$, taking into account the beaming factor. Again, we refer the reader to the paper by Yusifov and Küçük (2004) for further details.

Now, from the available catalogues at that time, Castro (2003) obtained that only 3% of the known pulsars present glitch phenomenon. The number of glitches in 25 yr amounted to 45. From Castro we obtain that the number of glitches per year per pulsar amounts to 2.6×10^{-3}.

Since we are considering the fact that the f-mode is capable of being excited during the glitch phenomenon we have

$$2.6 \times 10^{-3} \text{ events yr}^{-1} \text{ pulsar}^{-1}. \tag{3}$$

Finally, to obtain the number of events per year detectable by Schenberg, we use the results presented in figure 1.

Since the efficiency of generation of GW through the f-mode channel is not known, we present in figure 2 the event rate detectable by Schenberg as a function of $\varepsilon_{\mathrm{GW}}$. Note that for $\varepsilon_{\mathrm{GW}} > 10^{-8}$ (10^{-7}) we predict that one f-mode source could in principle be detected at SNR = 1 (SNR = 3) every year. It is worth noting that the number of pulsars used in the calculation of figure 2 takes into account the beaming correction.

Note that the event rate is critically dependent on the value of $\varepsilon_{\mathrm{GW}}$. In figure 2 one sees that if this parameter is too small the event rate is very small too.

However, the prediction appearing in figure 2 takes into account only the neutron stars in the form of pulsars with luminosity greater than 0.1 mJ y kpc^2 at 1400 MHz. The number of neutron stars in the Galaxy, pulsars or not, could well be a factor of a thousand, or even tens of thousands, greater.

Paczynski (1990), for example, estimates that there may exist $\sim 10^9$ neutron stars in the Galaxy. Other authors find similar figures, namely: Nelson *et al* (1995) and Walter (2001) argue that there may exist $\sim 10^8$–10^9 neutron stars in the Galaxy; whereas Timmes *et al* (1996),

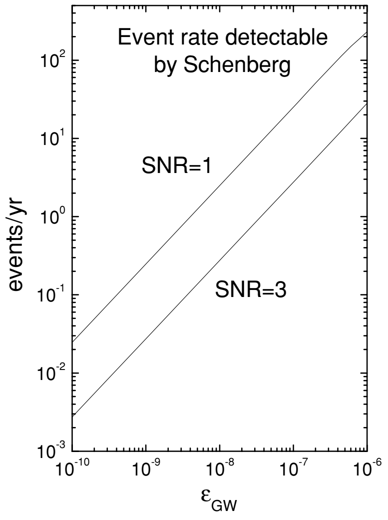

Figure 2. Number of events per year, i.e., number of f-mode unstable pulsars per year, as a function of $\varepsilon_{\rm GW}$, the efficiency of generation of GWs, detectable by Schenberg at SNR $= 1$ and 3, assuming that Schenberg's sensitivity to burst sources is $h \sim 10^{-20}$.

using models for massive stellar evolution coupled to a model for Galactic chemical evolution, obtained $\sim 10^{9}$ neutron stars in the Galaxy.

If the non-pulsar neutron stars could also be f-mode unstable, this means that the event rate detectable by Schenberg could be greatly enhanced. In particular, if the fraction of neutron stars and the fraction of pulsars, which are f-mode unstable, are similar this means that the event rate that could be detected by Schenberg would be a thousand, or even a few thousands, greater.

If this is the case, even though $\varepsilon_{\rm GW} \sim 10^{-10}$, Schenberg could detect ~10–100 events every year at SNR $= 3$.

4. Final remarks

Particular attention has been given here to the f-mode sources, because they could in principle be one of the most important candidates to be detected by the Schenberg antenna, with an event rate that could amount to several sources every year.

Since the interferometers are also sensitive to the GWs generated by f-mode neutron stars, it would be of interest to search for these sources with such detectors. Due to the fact that the sensitivity of the interferometers at 3 kHz (see, e.g., Shoemaker 2005) could well be similar to that of the Schenberg antenna, the event rate of both detectors could be similar.

Also, it is worth mentioning that the interferometers could probe a wider range of EOSs, as compared to the Schenberg antenna, since they are sensitive to broader GW frequency bands.

Finally, it is worth mentioning that Kokkotas *et al* (2001) show that detecting the f-mode, the EOS, the mass and the radius of the neutron stars will be strongly constrained. The reader should appreciate the reading of the paper by Kokkotas *et al*, who show in detail how the above-mentioned astrophysical information is obtained from the GW data.

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