INSTITUTE OF PHYSICS PUBLISHING CLASSICAL AND QUANTUM GRAVITY

Class. Quantum Grav. **21** (2004) S541–S544 PII: S0264-9381(04)68322-5

Wave polarizations for a beam-like gravitational wave in quadratic curvature gravity

E C de Rey Neto, J C N de Araujo and O D Aguiar

Instituto Nacional de Pesquisas Espaciais—Divisão de Astrofísica, Av. dos Astronautas 1758, São José dos Campos, 12227-010 SP, Brazil

E-mail: edgard@das.inpe.br, jcarlos@das.inpe.br and odylio@das.inpe.br

Received 29 August 2003
Published 5 February 2003
Online at stacks.iop.org/CQG/21/S541 (DOI: 10.1088/0264-9381/21/5/023)

Abstract

We compute analytically the tidal field and polarizations of an exact gravitational impulsive wave generated by a cylindrical beam of null matter of finite width and length in quadratic curvature gravity. We propose that this wave can represent the gravitational wave that keeps up with the high-energy photons produced in a gamma ray burst source.

PACS numbers: 04.20.Jb, 04.50.+h, 04.30.−w

1. Tidal field in a spacetime of a pp wave

The relative accelerations between particles located at nearby geodesics are determined by the geodesic deviation equation

$$\frac{D^2X^\mu}{d\tau^2} = -R^\mu{}_{\nu\gamma\delta}u^\nu X^\gamma u^\delta, \tag{1}$$

where τ is the proper time of one of the particles and the right-hand side represents the tidal force. By choosing an orthonormal tetrad $\{e_{\hat{a}}\}$ such that $e_{\hat{0}} = u$ is the 4-velocity of one of the test particles and $\{e_{\hat{i}}\}$, $i = 1, 2, 3$, are orthogonal spacelike unit 4-vectors, such that $e_{\hat{a}} \cdot e_{\hat{b}} \equiv g_{\mu\nu}e^\mu_{\hat{a}}e^\nu_{\hat{b}} = \eta_{\hat{a}\hat{b}} = \mathrm{diag}(-1, 1, 1, 1)$, we obtain $\ddot{X}^{\hat{0}} = R_{\mu\nu\gamma\delta}u^\mu u^\nu X^\gamma u^\delta = 0$ and

$$\ddot{X}^{\hat{i}} = -R^{\hat{i}}{}_{\hat{0}\hat{j}\hat{0}}X^{\hat{j}}, \tag{2}$$

where the overdots mean derivatives with respect to τ and $R^{\hat{i}}{}_{\hat{0}\hat{j}\hat{0}}$ are the projections of the Riemann tensor components on the tetrad frame $\{e_{\hat{a}}\}$.

Consider a gravitational pp wave given by [1]

$$ds^2 = -du\,dv + H(u, r, \phi)\,du^2 + dr^2 + r^2\,d\phi^2, \tag{3}$$

where $(\hbar = c = 1)$, $u = t - z$, $v = t + z$, and r and ϕ are polar coordinates in the plane perpendicular to the wave propagation direction. The nonvanishing accelerations are given by

$$\ddot{X}^{\hat{1}} = -(A_+ + A_\circ)X^{\hat{1}} + A_\times X^{\hat{2}}, \tag{4a}$$

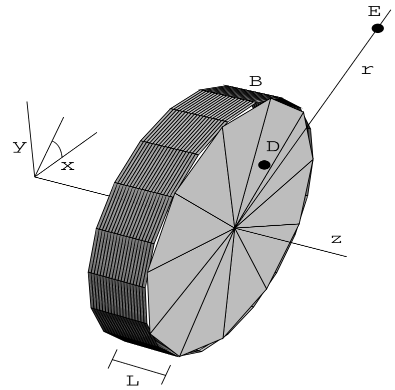


Figure 1. A cylindrical beam of radius B and length L of high-energy photons propagating with light velocity in the z direction. An observer D is crossed by the photon beam and an observer E lies outside the beam.

$$\ddot{X}^{\hat{2}} = +A_\times X^{\hat{1}} - (-A_+ + A_\circ)X^{\hat{2}}, \tag{4b}$$

where

$$A_\circ = \frac{1}{8}\nabla_\perp^2 H$$
$$A_+ = \frac{1}{8}\left(\frac{H_{,\phi\phi}}{r^2} + \frac{H_{,r}}{r} - H_{,rr}\right), \qquad A_\times = \frac{1}{4}\left(\frac{H_{,r\phi}}{r} - \frac{H_{,\phi}}{r^2}\right). \tag{5}$$

The comma stands for partial derivatives and $\nabla_\perp^2$ is the Laplacian in the transverse plane [2]. All patterns are transverse. The quantities A_+ and $A_\times$ generate helicity-2 polarization patterns shifted by $\pi/4$ while $A_\circ$ produces a helicity-0 pattern. We apply equations (4) only at large distances from the massive radiating bodies.

2. Gravitational wave generated by a cylindrical beam of photons

The quadratic gravity equation for the spacetime (3) becomes

$$-\tfrac{1}{2}\left[\beta\nabla_\perp^4 + \nabla_\perp^2\right]H(u, r, \phi) = 8\pi G T_{uu}, \tag{6}$$

where G is Newton's gravitational constant and β is the coupling parameter of the Ricci squared term in the gravitational action [3]. Consider a cylindrical beam of photons travelling in the z direction with constant energy density ϱ_0, radius B and length $L = c(t_F - t_I)$, where $t_F - t_I \equiv \delta t > 0$ is burst duration (figure 1). We have

$$T_{uu} = \varrho_0 \Theta(u - u_I)\Theta(u_F - u)\Theta(B - r), \tag{7}$$

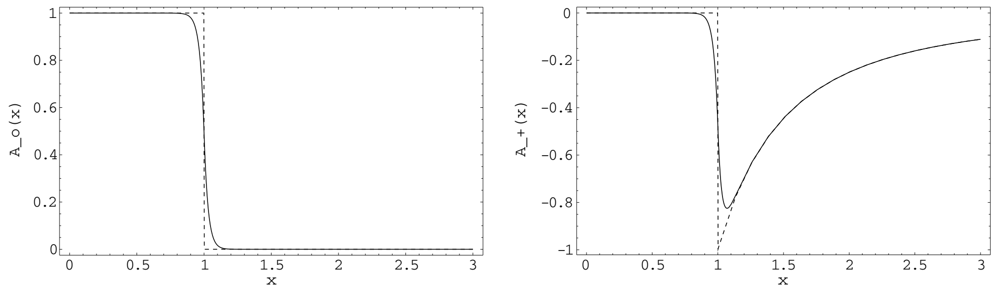
where $\Theta(x)$ is the Heaviside step function, $u_I = t_I - z, u_F = t_F - z$ and ϱ_0 is the energy density of the beam.

The solution of (6) with the source (7) is given by

$$H(x, u) = h(x) f(u), \tag{8}$$

where $f(u)$ is 1 if $u_I < u < u_F$ and 0 otherwise,

$$h(x) = \kappa\{[4BbK_1(b_0)I_0(x) - r^2 - 4b^2]\Theta[b(b_0 - x)]$$
$$- [2B^2 \ln(x) + B^2 + 4BbI_1(b_0)K_0(x)]\Theta[b(x - B_0)]\}, \tag{9}$$



(a) (b)

Figure 2. The profiles of $A_\circ$ and A_+. The solid curves represent the solutions to quadratic gravity and the dashed ones represent the solutions to Einstein gravity. The units are such that $\kappa/2 = 1$. We set a large value of b to obtain a better visualization of the quadratic curvature effect.

$\kappa = 4\pi G\varrho_0$, $x \equiv r/b$, $b_0 = B/b$, K_ν and I_ν are modified Bessel functions and $b \equiv (-\beta)^{(1/2)}$ [4]. We assume that $\beta < 0$ since for $\beta > 0$ it is known that there is no acceptable Newtonian limit for the nonrelativistic gravitational potential between point masses. The solution (8) implies that spacetime is flat for $u < u_I$ and $u > u_F$ and curved for $u_I < u < u_F$. The quantities (5) become

$$A_\circ = -\frac{\kappa}{2}\{[b_0K_1(b_0)I_0(x) - 1]\Theta[b(b_0 - x)] - b_0I_1(b_0)K_0(x)\Theta[b(x - b_0)]\}f(u), \tag{10}$$

$$A_+ = -\frac{\kappa}{2}\left\{b_0K_1(b_0)I_2(x)\Theta[b(b_0 - x)] + \left[b_0^2/x^2 - b_0I_1(b_0)K_2(x)\right]\Theta[b(x - b_0)]\right\}f(u) \tag{11}$$

and $A_\times = 0$, since $H_{,\phi} = 0$. In figure 2 we compare the profiles of A_0 and A_+ as functions of x in Einstein and quadratic gravity.

For a distant GRB progenitor, we can roughly approximate the energy density by

$$\varrho_0 \simeq \frac{E}{4\pi z^2 c\delta t}, \tag{12}$$

where z is the distance to the GRB source and E is the burst energy. If $r < B$, $T_{uu} = \varrho_0 f(u)$; there is a radiating field of nongravitational energy. If $r > B$, there are no radiating fields and $T_{uu} = 0$.

3. Effect on geodetic test particles

Consider observers D and E which measure the relative accelerations between test particles at great distances from the GRB progenitor, such that ϱ_0 is given by (12). The region $(r < B, x < b_0)$ is not a pure vacuum since $T_{uu} = \varrho_0 f(u)$. Therefore, there is a helicity-0 polarization pattern in addition to the helicity 2 [5, 6]:

$$A_\circ = \frac{\kappa}{2}[1 - b_0K_1(b_0)I_0(x)]f(u) \qquad \text{and} \qquad A_+ = -\frac{\kappa}{2}b_0K_1(b_0)I_2(x)f(u). \tag{13}$$

The quadratic gravity contributions are negligible with respect to Einstein gravity contributions unless $r \simeq B$.

For the observers E $(r > B, x < b_0)$,

$$A_\circ = \frac{\kappa}{2}b_0I_1(b_0)K_0(x)f(u) \qquad \text{and} \qquad A_+ = -\frac{\kappa}{2}\left[\frac{b_0^2}{x^2} + b_0I_1(b_0)K_2(x)\right]f(u). \tag{14}$$

Here, $A_\circ$ comes only from quadratic curvature interactions because there are no radiating fields in this region. We must not worry about the appearance of the helicity-0 component in (13), since this is the expected result when radiating fields are present [5, 6].

The greatest amplitude of the effect of the gravitational wave on geodetic test particles near the Earth is proportional to $2\pi G\varrho_0(\delta t)^2 \sim 10^{-38}$ (in dimensionless units) for a flux of $\sim 10^{-2}$ erg cm^{-2} s^{-1} and $\delta t \sim 10$ s, typical values for a GRB.

4. Summary and conclusions

We compute an exact pp wave solution to quadratic gravity equations with a cylindrical beam of photons as a source. We propose that this model can represent approximately a gravitational wave accompanying a beam of high-energy photons emitted in a GRB. By considering the geodesic deviations far from the GRB source we show that, for an observer that is crossed by the beam ($r < R$) during the interval δt, the helicity-2 polarization pattern is given only by the quadratic curvature effects and is negligible unless the observer is located at $r \simeq B$. This observer must see a helicity-0 pattern in the relative accelerations of test particles even in GR gravity. This result does not conflict with the GR expectations, since this observer is crossed by the photon beam and GW at the same time. An observer that is not crossed by the beam sees only a helicity-2 pattern which decreases with the square of the distance from the beam axis. The magnitude of the effect of the GW produced by the photon beam at the Earth on geodetic test particles is obviously very small.

Acknowledgments

We want to thank the Brazilian agency FAPESP for financial support (grants 00/10374-5, 98/13468-9 and 03/04342-1) and CNPq (grants 300619/92-8 and 304.666/02-5).

References

[1] Kramer D, Stephani H, Herlt E and MacCallum M 1980 *Exact Solutions of Einstein's Field Equations* (London: Cambridge University Press) p 233
[2] de Rey Neto E C 2003 *Preprint* gr-qc/0309128 (2003 *Phys. Rev.* D **68** at press)
[3] Buchdahl H A 1983 *J. Phys. A: Math. Gen.* **16** 1441
[4] de Rey Neto E C, de Araujo J C N and Aguiar O D 2003 *Class. Quantum Grav.* **20** 1479–87
[5] Kutschera M 2003 *Mon. Not. R. Astron. Soc.* **345** L1–5
[6] Podolský J and Ortaggio M 2003 *Class. Quantum Grav.* **20** 1685–701