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Gravitational wave background from stellar black holes in a scenario of structure formation

J C N de Araujo[1], **O D Miranda**[2] **and O D Aguiar**[1]

[1] Instituto Nacional de Pesquisas Espaciais—Divisão de Astrofísica, Av. dos Astronautas 1758, São José dos Campos, 12227-010 SP, Brazil
[2] Instituto Tecnológico de Aeronáutica—Departamento de Física, Praça Marechal Eduardo Gomes 50, São José dos Campos, 12228-900 SP, Brazil

E-mail: jcarlos@das.inpe.br, oswaldo@fis.ita.br and odylio@das.inpe.br

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Abstract

We study the generation of a stochastic gravitational wave (GW) background produced from a population of core-collapse supernovae, which form black holes in a scenario by Springel and Hernquist, who employed hydrodynamic simulations of structure formation in a ΛCDM cosmology. In addition, and in contrast to previous studies, we consider the effect of taking into account different values of the α parameter, the fraction of the progenitor mass which forms the black hole. We then study the detectability of this GW background and discuss what astrophysical information could be obtained from a positive, or even a negative detection of such a putative background.

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1. Introduction

Springel and Hernquist (2003; hereafter SH) obtain the history of star formation from hydrodynamic simulations of structure formation in ΛCDM cosmology. They study the history of cosmic star formation from the 'dark ages', at redshift $z \sim 20$ to the present. They take into account besides gravity and ordinary hydrodynamics, radiative heating and cooling of gas, supernova feedbacks and galactic winds. It is worth mentioning that the story of star formation they obtain is consistent with observations. It is important to bear in mind, however, that nowadays observations give information from a redshift around $z \sim 5$, at most. In the future, however, with the next generation space telescope (NGST) it will be possible to trace the cosmic star formation rate to $z \gtrsim 20$ (see, e.g., Mackey *et al* 2003).

In the present paper we study the generation of a background of gravitational waves (GWs) produced in the formation of black holes, with a Salpeter IMF, in the SH scenario. We also consider the role of the α parameter, the fraction of the progenitor mass which forms the remnant black hole. We then discuss what conclusions would be drawn whether (or not) the stochastic background studied here is detected by the forthcoming GW observatories such as LIGO and VIRGO.

In the following section we describe how to calculate the background of GWs produced during the formation of stellar black holes in the SH scenario. We then discuss the numerical results and the detectability of this putative GW background.

2. Gravitational wave production from the Springel and Herniquist stellar black-hole formation epoch and its detectability

Following de Araujo *et al* (2000) (see also de Araujo *et al* 2002), the dimensionless amplitude, $h_{\rm BG}$, of the stochastic GW background produced by gravitational collapses that lead to black holes is given by

$$h_{\rm BG}^2 = \frac{1}{\nu_{\rm obs}} \int h_{\rm BH}^2 \, {\rm d}R_{\rm BH}, \qquad (1)$$

where ${\rm d}R_{\rm BH}$ is the differential rate of black-hole formation, $h_{\rm BH}$ is the dimensionless amplitude produced by the collapse to a black hole that generates at the present time a signal with frequency $\nu_{\rm obs}$.

The dimensionless amplitude produced by the collapse of a star, or star cluster, to form a black hole is (Thorne 1987)

$$h_{\rm BH} = \left(\frac{15}{2\pi}\varepsilon_{\rm GW}\right)^{1/2} \frac{G}{c^2}\frac{M_{\rm r}}{d_{\rm L}} \simeq 7.4 \times 10^{-20} \varepsilon_{\rm GW}^{1/2} \left(\frac{M_{\rm r}}{M_\odot}\right)\left(\frac{d_{\rm L}}{1~{\rm Mpc}}\right)^{-1}, \qquad (2)$$

where $\varepsilon_{\rm GW}$ is the efficiency of generation of GWs, $M_{\rm r}$ is the mass of the black-hole remnant, and $d_{\rm L}$ is the luminosity distance to the source.

The collapse of a star to a black hole produces a signal with frequency (Thorne 1987)

$$\nu_{\rm obs} = \frac{1}{5\pi M_{\rm r}}\frac{c^3}{G}(1+z)^{-1} \simeq 1.3 \times 10^4~{\rm Hz}\left(\frac{M_\odot}{M_{\rm r}}\right)(1+z)^{-1}, \qquad (3)$$

where the factor $(1+z)^{-1}$ takes into account the redshift effect on the emission frequency, that is, a signal emitted at frequency $\nu_{\rm e}$ at redshift z is observed at frequency $\nu_{\rm obs} = \nu_{\rm e}(1+z)^{-1}$.

For the differential rate of black-hole formation we have

$${\rm d}R_{\rm BH} = \dot\rho_\star(z)\frac{{\rm d}V}{{\rm d}z}\phi(m)\,{\rm d}m\,{\rm d}z, \qquad (4)$$

where $\dot\rho_\star(z)$ is the star formation rate (SFR) density, $\phi(m)$ is the IMF and ${\rm d}V$ is the comoving volume element.

Here the Salpeter IMF is adopted, namely, $\phi(m) = Am^{-(1+x)}$—where A is the normalization constant, m is the progenitor mass and $x = 1.35$ (our fiducial value). The remnant and the progenitor masses are related to $M_{\rm r} = \alpha m$. We assume, as usual, that the progenitor masses of the black holes range from 25–125$M_\odot$ (see, e.g., de Araujo *et al* 2002).

For the SFR density we consider the one derived by SH, namely

$$\dot\rho_\star(z) = 0.15\frac{14\exp\left[\frac{3}{5}(z-5.4)\right]}{5+3\exp\left[\frac{14}{15}(z-5.4)\right]}\,M_\odot\,{\rm yr}^{-1}\,{\rm Mpc}^{-3}. \qquad (5)$$

SH consider a ΛCDM model with the following parameters: $\Omega_{\rm M} = 0.3$, $\Omega_\Lambda = 0.7$, Hubble constant $H_0 = 100\,h$ km s^{-1} Mpc^{-1} with $h = 0.7$, $\Omega_{\rm B} = 0.04$, and a scale-invariant primordial power spectrum with index $n = 1$, normalized to the abundance of rich galaxy clusters at present ($\sigma_8 = 0.9$).

Table 1. For different values of α we present the S/N for pairs of LIGO I, II and III ('first', 'enhanced' and 'advanced', respectively) observatories for one year of observation. Note that an efficiency of generation $\varepsilon_{\mathrm{GW}_{\max}} = 7 \times 10^{-4}$ is assumed. Also shown are the GW frequency bands, $\Delta\nu$, for different values of α.

		S/N		
α	$\Delta\nu$ (Hz)	LIGO I	LIGO II	LIGO III
0.1	50–5200	6.7×10^{-6}	2.9×10^{-4}	6.0×10^{-4}
0.2	25–2600	1.6×10^{-4}	1.6×10^{-2}	5.4×10^{-2}
0.3	16–1700	6.3×10^{-4}	9.1×10^{-2}	0.40
0.4	12–1300	1.5×10^{-2}	0.24	1.4
0.5	9.9–1000	3.0×10^{-3}	0.49	3.7
0.6	8.2–860	5.0×10^{-3}	0.85	7.4
0.7	7.1–740	7.7×10^{-3}	1.4	13
0.8	6.2–650	1.1×10^{-2}	2.0	20
0.9	5.5–580	1.4×10^{-2}	2.8	29
1.0	5.0–520	1.7×10^{-2}	3.9	40

To calculate h_{BG} it is necessary to set values for two important parameters. Firstly, $\varepsilon_{\mathrm{GW}}$, the efficiency of production of GWs, whose distribution function is unknown. Thus, we have parametrized our results in terms of its maximum value, namely, $\varepsilon_{\mathrm{GW}_{\max}} = 7 \times 10^{-4}$, which is obtained from studies by Stark and Piran (1986) who simulated the axisymmetric collapse of a rotating star to a black hole.

Secondly, to calculate h_{BG} we still need to set a value for α. Note that α may depend sensitively on the metallicity: the lower the value of Z, the higher the remnant masses and the less ejected material relative to $Z_{\odot}$ stars.

It is worth noting that the background predicted in the present study cannot be detected by single forthcoming interferometric detectors, such as VIRGO and LIGO (even by advanced ones). However, it is possible to correlate the signal of two or more detectors to detect the background that we propose exists.

To assess the detectability of a GW signal, one must evaluate the signal-to-noise ratio (S/N), which for a pair of interferometers is given by (see, e.g., Allen 1997)

$$(\mathrm{S/N})^2 = \left[\left(\frac{9H_0^4}{50\pi^4}\right) T \int_0^\infty \mathrm{d}\nu \, \frac{\gamma^2(\nu)\Omega_{\mathrm{GW}}^2(\nu)}{\nu^6 S_h^{(1)}(\nu) S_h^{(2)}(\nu)}\right], \tag{6}$$

where $S_h^{(i)}$ is the spectral noise density, T is the integration time, Ω_{GW} is the density parameter in GWs and $\gamma(\nu)$ is the overlap reduction function, which depends on the relative positions and orientations of the two interferometers. Here we consider, in particular, the LIGO interferometers, whose spectral noise densities have been taken from a paper by Owen *et al* (1998). The closure energy density is given by

$$\Omega_{\mathrm{GW}} = \frac{4\pi^2}{3H_0^2} \nu_{\mathrm{obs}}^2 h_{\mathrm{BG}}^2 \tag{7}$$

(see, e.g., de Araujo *et al* 2000).

In table 1 we present the S/N for one year of observation for $\varepsilon_{\mathrm{GW}_{\max}} = 7 \times 10^{-4}$ and different values of α, for the three LIGO interferometer configurations considered by Owen *et al* (1998). We also present in table 1 the GW frequency bands, $\Delta\nu$, for the different values of α.

Note that $\Delta\nu$ is very sensitive to the value of α. Note that the frequency band is directly related to the remnant mass, the black-hole mass, through equation (3). Then, once the GW background is observationally obtained, namely, h_{BG} versus ν_{obs}, it is possible to obtain the GW frequency band and consequently the black-hole mass range.

Before proceeding, it is worth mentioning that a relevant question is whether the background is continuous or not. The duty cycle indicates if the collective effect of the bursts of GWs generated during the collapse of a progenitor star generates a continuous background. The duty cycle is $\gg 1$ for all the models studied here for which the $\mathrm{S/R} > 1$, therefore the background is continuous in these cases.

Note that for the 'initial' LIGO (LIGO I) there is no hope of detecting the GW background we propose here. For the 'enhanced' LIGO (LIGO II) there is some possibility of detecting the background, since $\mathrm{S/N} > 1$, if $\varepsilon_{\mathrm{GW}}$ is around the maximum value and $\alpha > 0.6$. Even if the LIGO II interferometers cannot detect such a background, it will be possible to constrain the efficiency of GW production and α.

The prospect for the detection with the 'advanced' LIGO (LIGO III) interferometers is much more optimistic, since the S/N for $\alpha > 0.4$ is significantly greater than unity. Only if the value of $\varepsilon_{\mathrm{GW}}$ was significantly lower than the maximum value would the detection not be possible. In fact, the S/N is critically dependent on this parameter. We have adopted here the maximum value as a reference, but if its actual value is much less than this value the S/N could be lower than unity for all the models studied here, even for a LIGO III pair. Let us think of what occurs with other compact objects, namely, neutron stars, to see if we can learn something from them. Hot and rapidly rotating neutron stars can lose angular momentum to gravitational radiation via the so-called r-mode instability (Andersson 1998). This could explain why all known young neutron stars are relatively slow rotators. Black holes could have had a similar history, i.e., they could have been formed rapidly rotating and lost angular momentum to gravitation radiation via their quasi-normal modes. If this was the case, the value of $\varepsilon_{\mathrm{GW}}$ could be near the maximum.

Our results also show that the S/N is sensitive to variations of α. The larger the α, the lower are the GW frequencies and the higher is h_{BG}, and since the best window for detection is around $0 < \nu < 64$ Hz, the S/N is higher.

Even if α is not known beforehand, it is possible to impose a constraint on its values. For example, if one found from GW observations that the GW frequency band was 10–1000 Hz, one would obtain (using equation (3)) that $\alpha \simeq 0.5$.

We leave the detailed study of the role of the variation of the several parameters considered here to another paper to appear elsewhere.

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