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Temperature fluctuations of the cosmic microwave background radiation: A case of non-extensivity?

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Abstract

Temperature maps of the cosmic microwave background (CMB) radiation, as those obtained by the Wilkinson microwave anisotropy probe (WMAP), provide one of the most precise data sets to test fundamental hypotheses of modern cosmology. One of these issues is related to the statistical properties of the CMB temperature fluctuations. We analysed here the WMAP data and found that the distribution of the CMB temperature fluctuations $P^{\mathrm{CMB}}(\Delta T)$ can be quite well fitted by the anomalous temperature distribution emerging within non-extensive statistical mechanics. This theory is based on the non-extensive entropy $S_q \equiv k\{1 - \int dx\,[P_q(x)]^q\}/(q-1)$, with the Boltzmann–Gibbs expression as the limit case $q \to 1$. For the frequencies investigated ($\nu = 40.7$, 60.8, and 93.5 GHz), we found that $P^{\mathrm{CMB}}(\Delta T)$ is well described by $P_q(\Delta T) \propto 1/[1 + (q-1)B(\nu)\Delta T^2]^{1/(q-1)}$, with $q = 1.045 \pm 0.005$, which indicate, at the 99% confidence level, that Gaussian temperature distributions $P^{\mathrm{Gauss}}(\Delta T) \propto e^{-B(\nu)\Delta T^2}$, corresponding to the $q \to 1$ limit, do not properly represent the CMB temperature fluctuations measured by WMAP.

Keywords: Cosmic background radiation: Temperature fluctuations; Non-extensive statistical mechanics; Non-Gaussian distributions

Cosmic microwave background (CMB) radiation is formed by the leftover photons from the early universe, when matter and radiation were coupled in thermal equilibrium. With the expansion of the universe, these photons decoupled and spread out freely, basically conserving their primordial features. In the early 90s, the far infrared absolute spectrophotometer, on board the cosmic background explorer (COBE) satellite, proved that this radiation was in thermal equilibrium by measuring its Planckian spectrum, with the temperature $T_0 = 2.725 \pm 0.002$ K [1]. Although the accuracy of these data (within limits as tight as 0.03% in the frequency range 60–600 GHz) left no doubt about the past thermal equilibrium state of the CMB, it is still possible that such Planck law derived within Boltzmann–Gibbs statistics does not exactly describe this radiation and may instead obey a generalized expression. A plausible distribution could be that obtained within non-extensive statistical mechanics (for a review on this subject see, e.g. [2]).

Observations with another instrument on board COBE, the differential microwave radiometer (COBE-DMR) [3], detected for the first time that the CMB exhibits tiny variations around T_0, termed CMB temperature fluctuations ΔT, at the level of one part in 10^5 on large angular scales ($\sim 7°$). Since the standard inflationary cosmology predicts that these temperature fluctuations should be isotropic and Gaussian random, the COBE-DMR data motivated a number of analyses, although not so accurate due to the large angular resolution of the data, to test the statistical properties of the CMB (see, e.g. [4]).

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Recently, very precise and high angular resolution data from the Wilkinson microwave anisotropy probe (WMAP) [5] confirmed the existence of the CMB temperature fluctuations. The WMAP satellite observes the microwave sky in five frequency bands, K, Ka, Q, V, and W, centered on the frequencies 22.8, 33.0, 40.7, 60.8, and 93.5 GHz, respectively. The corresponding CMB maps released by the WMAP team are pixelized in the HEALPix scheme [6] with a resolution parameter $N_{\text{side}} = 512$, which means that the celestial sphere is covered with 3, 145, 728 equal-area pixels.

The WMAP data have renewed concerns over the CMB statistical properties, and considerable analyses of the Gaussian hypothesis have been done [7]. Clearly, the study of such hypothesis must take into account the possibility that deviations from Gaussianity may have non-cosmological origins such as unsubtracted foreground contamination, instrumental noise, and/or systematic effects [8]. But they may also have cosmological origin, as being, for instance, the effect of cosmic strings on CMB [9]. In a variety of analyses using different mathematical tools, and including foreground cleaning processes aimed to eliminate possible non-Gaussian contaminations, many evidences regarding deviations from Gaussianity—and also from statistical isotropy—in the WMAP CMB data have been recently reported [10] (see also [11] and references therein).

In what follows we shall perform the statistical analysis of the Q, V and W maps, after the application of a exclusion mask to eliminate known foregrounds, in order to determine how much their distributions of temperature fluctuations deviate, if they do, from the Gaussian distribution. Then we discuss the possible account of such distributions according to the non-Gaussian temperature distribution emerging within non-extensive statistical mechanics. Definitely, the statistical significance of our results shall be supported by the analysis of 10 000 Monte Carlo CMB maps. These Monte Carlo maps are stochastic realizations of the WMAP best-fitting angular power spectrum of the ΛCDM model [5]. They are produced through randomized multipole components which are obtained, within the cosmic variance limits, using the map-making code SYNFAST from HEALPix [6]. Finally, using a χ^2 estimator test we assess the confidence level in the fits of WMAP data compared to the fits of Monte Carlo CMB maps.

Below, we briefly introduce the basics of non-extensive statistical mechanics (see [12] for various applications). The probability distribution $P_q(x)$, as a function of the variable x, results from the optimization of the q-entropy defined by [2]

$$S_q \equiv k \frac{1 - \int dx\,[P_q(x)]^q}{(q-1)}, \tag{1}$$

with the constraints

$$\int dx\, P_q(x) = 1, \tag{2}$$

$$\frac{\int dx\, x[P_q(x)]^q}{\int dx\,[P_q(x)]^q} = c_1, \tag{3}$$

$$\frac{\int dx\, x^2[P_q(x)]^q}{\int dx\,[P_q(x)]^q} = c_2. \tag{4}$$

From this, we straightforwardly obtains

$$P_q(x) = \frac{e_q^{-\lambda_{1q}(x-c_1)-\lambda_{2q}(x^2-c_2)}}{\int dx\, e_q^{-\lambda_{1q}(x-c_1)-\lambda_{2q}(x^2-c_2)}}, \tag{5}$$

where λ_{1q} and λ_{2q} are related to the Lagrange multipliers, and

$$e_q^z \equiv \left[1 + (1-q)z\right]^{1/(1-q)}, \quad \text{for } \left[1 + (1-q)z\right] \geqslant 0, \tag{6}$$

while $e_q^z = 0$ otherwise. Note that this distribution function P_q is the solution of a non-linear Fokker–Planck equation [13]. Eq. (5) can be rewritten as follows

$$P_q(x) = \frac{e_q^{-B_q(x-x_0)^2}}{\int dx\, e_q^{-B_q(x-x_0)^2}} = A_q e_q^{-B_q(x-x_0)^2}, \tag{7}$$

where the values of B_q and x_0 are related to λ_{1q} and λ_{2q}; A_q is the normalization constant obtained in such a way that Eq. (2) is satisfied.

We shall now use this non-extensive distribution to account for the CMB temperature fluctuations distribution of WMAP data. In this case we have $x = T$, $x_0 = T_0$, and $B_q = B(\nu)$. In what follows we analyze the data by plotting the normalized number of pixels versus $(\Delta T/\sigma_\nu)^2$ because in these kind of plots a Gaussian distribution fits a straight line, becoming easy to recognize departures from Gaussianity (i.e. linearity). Strictly speaking $B_q = B_q(\nu)$, however, for a clear comparison between the Gaussian (linear) fit and non-extensive distributions in these plots we assume that not only both distributions are equal at the initial point (i.e. $\Delta T = 0$), but also that the best-fitting tangent line at that point has equal slope for both distributions, which implies that $B_q = B(\nu)$ (this result can, of course, be analytically obtained by taking the $\log_{10}$ in Eqs. (8) and (9), deriving with respect to ΔT^2, and finally equating both derivatives for the point $\Delta T = 0$).

In such a case, the best-fitting of the CMB data through the non-extensive distributions is obtained only adjusting the q-parameter. Thus,

$$P_q(\Delta T) = A_q e_q^{-B(\nu)\Delta T^2}, \tag{8}$$

where we wrote $P_q(\Delta T)$ instead of $P_q(T)$ to be clear that our analysis is dealing with the statistics of the temperature fluctuations. In the limit $q \to 1$, we recover the Gaussian distribution

$$P_q \to P^{\text{Gauss}} = A e^{-B(\nu)\Delta T^2}, \tag{9}$$

where $A \equiv 1/(\sigma_\nu\sqrt{2\pi})$, $B(\nu) \equiv 1/(2\sigma_\nu^2)$, and σ_ν^2 is the variance of the Gaussian distribution. For a comparison of the distributions P_q with P^{Gauss}, in our analyses we assume $A_q = A$.

The signal measured at any pixel in the microwave sky is form by the following components:

$$T_{\text{pixel}} = T_{\text{foregrounds}} + T_{\text{noise}} + T_{\text{CMB}}, \tag{10}$$

corresponding to foreground signals, the noise from the instruments, and the CMB temperature fluctuations, respectively. Foreground contributions are expected around the Galactic plane and from point sources. For this, a set of masks (termed Kp0, Kp2, etc.) are provided by the WMAP team [5] which

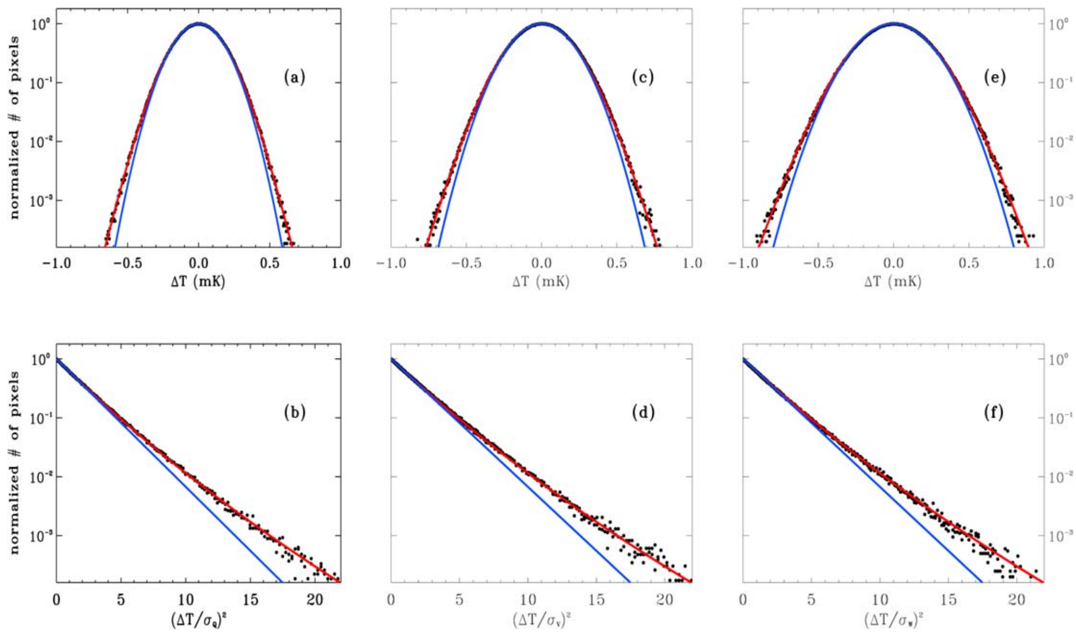

Fig. 1. Fits to CMB temperature fluctuations measured by WMAP in bands Q, V, and W (after using the Kp0 mask) applying a Gaussian distribution (blue curve) and a non-extensive function (red curve). In (a), (c), and (e) we plotted the normalized number of pixels versus ΔT, while in (b), (d), and (f) we plotted the normalized number of pixels versus $(\Delta T/\sigma_\nu)^2$, respectively. The χ^2 best-fit for the distributions P_q gives $q = 1.045$, with $\sigma_Q = 140.58$ μK, $\sigma_V = 163.52$ μK, and $\sigma_W = 190.35$ μK, respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

allow for the selective exclusion of portions of the sky basically due to contaminating radiative processes from our galaxy, and including a 0.6 degree radius exclusion area around known point sources in the celestial sphere. The application of a mask to a CMB map simply means that, for a given pixel, a mask value of zero implies that such pixel is excluded; a value of one means that the pixel is accepted.[2] In this way, one is left with CMB signal plus the Gaussian signal from the instrument noise (actually the signal noise is Gaussian *per* observation [9,14]).

Thus, we have to consider these effects by defining (e.g. [9]) the variance of the total signal $T_{\rm pixel}$ as

$$\sigma_\nu^2 \equiv \sigma_{\rm pixel}^2 = \frac{\sigma_0^2}{n_i} + \sigma_{\rm CMB}^2, \tag{11}$$

where n_i is the number of observations for the ith pixel, σ_0^2 is the noise variance *per* observation, and $\sigma_{\rm CMB}^2$ is the variance of the CMB temperature fluctuations. The mean contribution of the instrumental noise can be estimated by considering the effect of the different number of observations for each pixel (see Ref. [14], p. 16). Thus, given a CMB map, this can be done with the effective variance due to noise

$$\sigma_{\rm noise}^2(\nu) = \frac{\sigma_0^2 \sum_{i=1}^{N}(1/n_i)}{N}, \tag{12}$$

[2] The Q, V, and W maps here analysed were already corrected by the WMAP team for the Galactic foregrounds (synchrotron, free–free, and dust emission) using the 3-band, 5-parameter template fitting method described in [15]. However, the foreground removal is only applicable to regions outside the Kp2 mask (which cuts 15.0% of the sky data). For this, the Kp2 mask, or preferably the more severe Kp0 mask (which excludes 23.2% of the sky data), should be applied for the statistical analysis of the CMB maps.

where σ_0^2 is the variance *per* observation characteristic of the instrument (radiometer), n_i is the effective number of observations for the ith pixel, and N is the total number of pixels considered in the analysis of the map. For the Q, V and W maps we obtained $\sum(1/n_i) = 7\,423.90$, $5\,465.98$, and $1\,816.96$, respectively; moreover, the number of pixels analysed N is equal for the three maps $N_Q = N_V = N_W = 2\,414\,705$. In other words, the effective noise variances $\sigma_{\rm noise}^2(\nu)$ and the variances σ_ν^2 leads to the CMB variance

$$\sigma_{\rm CMB}^2 = \sigma_\nu^2 - \sigma_{\rm noise}^2(\nu). \tag{13}$$

Although both σ_ν^2 and $\sigma_{\rm noise}^2(\nu)$ depend on the map under analyses, their difference is independent of the map, in other words, if our treatment of instrumental noise is correct, the CMB variance $\sigma_{\rm CMB}^2$ should be the same for the three CMB maps under investigation (Q, V, and W).

As previously reported (see Section 2.2 and Fig. 2 in Ref. [9]) the WMAP CMB temperature distribution does not fully obey a Gaussian temperature distribution

$$P^{\rm Gauss}(\Delta T) = \frac{1}{\sigma_\nu\sqrt{2\pi}} e^{-(1/2\sigma_\nu^2)\Delta T^2}. \tag{14}$$

In fact, the deviation from a Gaussian distribution can be appreciated in Figs. 1(a), 1(c), and 1(e), for the Q, V, and W maps, respectively. In our analyses, the best-fit Gaussian to the data was obtained according to the χ^2/degree of freedom (dof) estimator test. Thus, the $\chi^2/200$ values for the Gaussian fits are 0.116, 0.275, and 0.167, for the Q, V and W maps, respectively, and the $\chi^2/200$ values for the non-extensive temperature distribution P_q are 0.00155, 0.00178, and 0.00216, for the Q, V and W maps, respectively. Analyses performed using 400 dof instead of 200 dof result in similar estimative values.

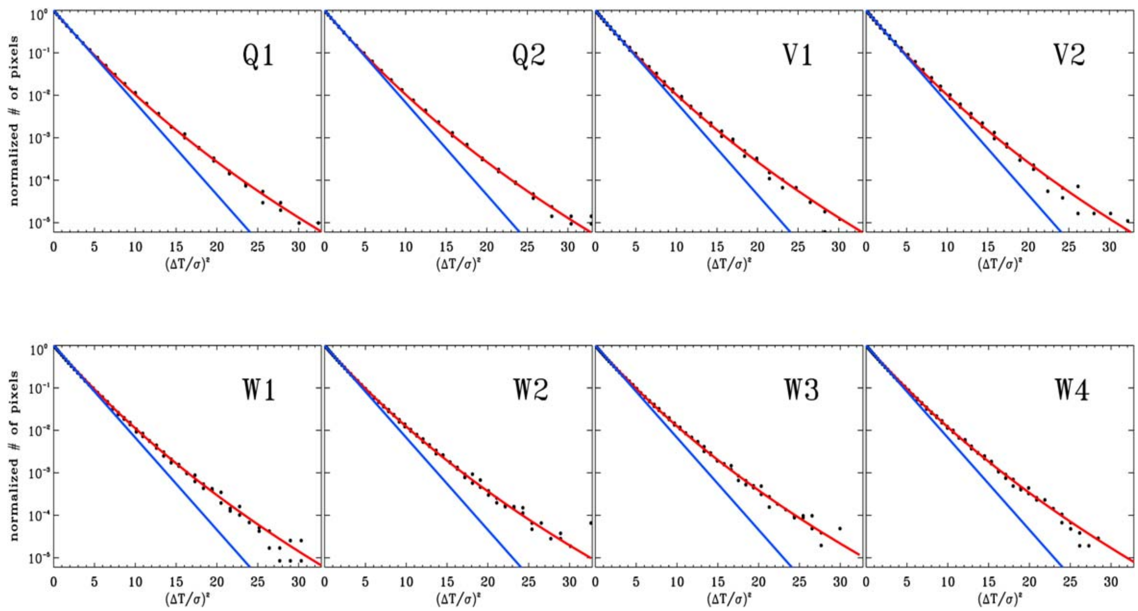

Fig. 2. Results of the analysis of WMAP data using the individual radiometer maps Q1, Q2, V1, V2, W1, W2, W3, and W4, after applying the Kp0 mask. In all plots the blue curve represents the best-fit Gaussian, according to the χ^2 estimator, while the red curve represents the best non-extensive distribution function. Each P_q distributions fits the corresponding data with a slightly different value of q, where the mean value of them is $q = 1.045 \pm 0.005$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

In Fig. 1 the blue lines correspond to the best-fit Gaussian with variances $\sigma_{\rm Q} = 140.58$ μK, $\sigma_{\rm V} = 163.52$ μK, and $\sigma_{\rm W} = 190.35$ μK, while the red lines correspond to the best non-extensive distribution fit with the same variances and $q = 1.045$. Once the variances σ_ν^2 have been determined through the χ^2 best-fit Gaussian temperature distribution, for each of the CMB maps, we use the effective noise variance $\sigma^2_{\rm noise}$ given in Eq. (12) again for each of the CMB maps, to calculate the CMB variance. Our results are $\sigma^2_{\rm CMB} = (68.77)^2$, $(69.34)^2$, $(68.81)^2$ μK^2, for the Q, V, and W maps, respectively, in excellent agreement with what is expected. This result validates the effective noise variance as representing the mean contribution of the instrumental noise. In Figs. 1(a), 1(c), and 1(e) we plotted the temperature distributions of the Q, V, and W maps, in the form $\log_{10}(P^{\rm CMB\ data})$, $\log_{10}(P^{\rm Gauss})$, and $\log_{10}(P_q)$ versus ΔT using the χ^2 best-fit Gaussian temperature distribution $P^{\rm Gauss}$ and also the best-fit non-extensive distributions P_q with $q = 1.045$. To clearly exhibit the non-Gaussian behavior of the WMAP data, in Figs. 1(b), 1(d), and 1(f), we plotted instead $\log_{10}(P^{\rm CMB\ data})$, $\log_{10}(P^{\rm Gauss})$, and $\log_{10}(P_q)$ versus $(\Delta T/\sigma_\nu)^2$, since linearity (blue curves) corresponds to the best-fit Gaussian distribution. Our results show that the distribution of the CMB temperature fluctuations does not obey a Gaussian distribution.

In order to strengthen our analysis, we removed fits to the WMAP foreground templates for the eight individual radiometer maps (corresponding to radiometers Q1, Q2, V1, V2, W1, W2, W3, and W4), after applying the Kp0 mask to avoid high Galactic latitude foregrounds. Our results are plotted in Fig. 2. As observed, these results were essentially the same as those shown in Fig. 1, that is, $q = 1.045 \pm 0.005$.

Then, we investigated the possibility that the discrepancies observed between WMAP data and Gaussian distributions, as evidenced in Figs. 1 and 2, occur just by chance. For this scope, we analysed a set of 10 000 Monte Carlo realizations of CMB Gaussian maps, and we found, at the 99% confidence level (CL), that the distributions of CMB temperature fluctuations measured by WMAP are not properly described by Gaussian temperature distributions.

We also investigated, through numerical simulations, the possibility that the instrumental noise could introduce non-Gaussian signals in the CMB maps, as expected. Instrument noise produces a random Gaussian signal (e.g. [9,14]), actually Gaussian *per* observation. Since pixels are observed a different number of times, the effect on the CMB map could be a significant non-Gaussian signal in the CMB temperature distribution, and clearly this possibility should be taken into account in the simulations. For this, to each of the 10 000 Monte Carlo CMB Gaussian maps we added a simulated non-stationary Gaussian radiometer noise, taking into account the actual number of observations (n_i) for each pixel in the maps. We consider the n_i data from the WMAP-W4 map [5]. We obtained $q = 1.005 \pm 0.01$ for the CMB plus non-stationary Gaussian noise simulated maps, which is obviously consistent with Gaussianity. A χ^2 estimator test shows that the $P_q(\Delta T)$ does not fit the simulated data as well as it does the WMAP data. The monotonic discrepancy behavior observed in Figs. 1(b), 1(d), and 1(f), which is well fitted by the non-extensive expression given in Eq. (7) due to a minimum χ^2 value, was observed in less than 1% of the simulations performed. Therefore we can rule out, at the 99% CL, the simulated radiometer noise as being the explanation for the non-Gaussianity observed in the WMAP temperature fluctuations.

In conclusion, we have shown that a Gaussian distribution is unsuitable, at the 99% CL, to properly represent the CMB temperature fluctuations measured by WMAP. Although the value of the q-parameter is close to 1, our analyses indicate that to consider these temperature fluctuations as being of Gaussian nature is not appropriate but it can be considered as a good approximation instead. Through the analysis presented here of very precise observational data, we conclude that the hypothesis of a non-extensive nature for the CMB temperature fluctuations is quite plausible.

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