

## ON THE LARGE-SCALE ANGULAR DISTRIBUTION OF SHORT GAMMA-RAY BURSTS

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### ABSTRACT

We investigate the large-scale angular distribution of the short gamma-ray bursts (SGRBs) from BATSE experiment, using a new coordinate-free method. The analyses performed take into account the angular correlations induced by the nonuniform sky exposure during the experiment, and the uncertainty in the measured angular coordinates. Comparing the large-scale angular correlations from the data with those expected from simulations using the exposure function, we find similar features. In addition, confronting the large-angle correlations computed from the data with those obtained from simulated maps produced under the assumption of statistical isotropy, we found that they are incompatible at 95% confidence level. However, such differences are restricted to the angular scales  $36^\circ$ – $45^\circ$ , which are likely to be due to the nonuniform sky exposure. This result strongly suggests that the set of SGRBs from BATSE are intrinsically isotropic. Moreover, we also investigated a possible large-angle correlation of these data with the supergalactic plane. No evidence for such large-scale anisotropy was found.

*Subject headings:* gamma rays: bursts — large-scale structure of universe — methods: statistical

*Online material:* color figures

### 1. INTRODUCTION

The apparent isotropy in the large-scale angular distribution of the gamma-ray bursts (GRBs) is a long-standing debate (Meegan et al. 1992; Briggs et al. 1996; Tegmark et al. 1996; Piran & Singh 1997; Metzger et al. 1997; Balázs et al. 1998; Mészáros et al. 2000). Since the first detection with the *VELA* satellite (Klebesadel et al. 1973), the origin of these highly energetic events has remained a challenge. Even if the origin of GRBs turns out to be extragalactic or cosmological, as suggested by current data (see, e.g., Piran 2005; Zhang & Mészáros 2004; Mészáros 2002, 2006 and references therein), this does not ensure that their distribution is isotropic. Up to now, no dominant anisotropies has been found in the angular distribution of GRBs. However, if detected, small anisotropic effects may reveal valuable information about their origin. In addition, the discovery of a large angular scale pattern in the sky distribution of GRBs may be useful to identify their sources by cross-correlating them with catalogs of cosmic objects, e.g., early-type galaxies, hard X-ray sources, etc. (Briggs et al. 1996; Tegmark et al. 1996; Piran 2005; Mészáros 2006).

The reported statistical analyses of the all-sky survey data from BATSE show that their large-scale angular distribution is consistent with isotropy (Piran 2005; Mészáros 2006), although aspects like observational artifacts have not been fully considered in some of these studies. It is well known that anisotropies with distinct origins manifest themselves on different angular scales and with different magnitudes. In this connection, it is reasonable to consider different approaches that can, in principle, provide information about multiple types of anisotropy, imprinted as angular correlation signatures (ACSs), that may be possibly present in the data.

Here we apply a new coordinates-free method to search for large-scale ACSs ( $\geq 18^\circ$ ) in a subset of the BATSE GRB data (Meegan et al. 2000), namely, the short GRBs, and then investigate their significance levels through the comparison with a large number of Monte Carlo maps. Such simulated maps were produced under similar conditions as the catalog under analysis, that is, taking into account the nonuniform sky exposure of BATSE and the uncertainty in the coordinates measurements. Furthermore,

for completeness, we also compare the ACSs of the catalog of GRBs with those corresponding to statistically isotropic Monte Carlo maps. Finally, we also investigated the possible large-scale angular correlation between the set of short GRBs and the supergalactic plane, in an attempt to search for likely host galaxies of these events (as recently suggested by Ghirlanda et al. 2006).

The outline of this paper is as follows: In § 2 we use GRBs data from the BATSE experiment to determine the short GRBs catalog to be investigated, and in § 3, we describe the method employed in such studies. The data analyses and results are shown in § 4, and finally in § 5 we formulate our conclusions.

### 2. THE SHORT GRBs FROM BATSE CATALOG

The physical analysis of GRBs utilizes their temporal and spectral properties (Fishman & Meegan 1995). Despite the different light curves observed in the spectra of GRBs, a useful parameter to classify them is the burst duration  $T_{90}$ , defined as the time interval during which 90% of the fluence is measured. The current BATSE catalog 4Bc (Meegan et al. 2000) contains 2702 events, whereas only 2037 GRBs have their parameter  $T_{90}$  measured (Meegan et al. 2000).

At first, the  $T_{90}$  value was used to divide the set of GRBs into two different subclasses: the short GRBs (SGRBs), with  $T_{90} < 2$  s, and the long GRBs, with  $T_{90} \geq 10$  s (Kouveliotou et al. 1993; Zhang & Mészáros 2004; Mészáros 2002, 2006). However, the use of this definition of SGRBs is instrument dependent and is susceptible to observational biases (Hakkila et al. 2007a). For this reason, one should consider in addition to the  $T_{90}$  criterion, the parameter  $HR_{3/21}$  (Mukherjee et al. 1998), which is defined as the 100–300 keV fluence divided by the 25–100 keV fluence of each GRB in BATSE 4Bc (Hakkila et al. 2007a, 2007b). Thus, the appropriate definition for a catalog of SGRBs is (Hakkila et al. 2007a, 2007b):  $C = \{516 \text{ events with } 2 \text{ s} \leq T_{90} < 4.7 \text{ s and } HR_{3/21} > 3\}$ .

With this information, and using a new coordinates-free method to be described in the next section, we perform a detailed analysis of the large-scale ACS present in the sky distribution of the SGRBs from BATSE.

### 3. THE 2PACF AND THE SIGMA-MAP ANALYSIS

Let  $\Omega_j^{\gamma_0} \equiv \Omega(\theta_j, \phi_j; \gamma_0) \in \mathcal{S}^2$  be a spherical cap region on the celestial sphere, of  $\gamma_0$  degrees of aperture, with vertex at the  $j$ th pixel,  $j = 1, \dots, N_{\text{caps}}$ , where  $(\theta_j, \phi_j)$  are the angular coordinates of the center of the  $j$ th pixel. Both, the number of spherical caps  $N_{\text{caps}}$  and the coordinates of their center  $(\theta_j, \phi_j)$  are defined using the HEALPix pixelization scheme (Górski et al. 2005). The spherical caps are such that their union completely covers the celestial sphere  $\mathcal{S}^2$ .

Let  $\mathcal{C}^j$  be the catalog of cosmic objects located in the  $j$ th spherical cap  $\Omega_j^{\gamma_0}$ . The 2PACF of these objects (Chen & Hakkila 1998; Padmanabhan 1993), denoted as  $\Upsilon_j(\gamma_i; \gamma_0)$ , is the difference between the normalized frequency distribution and that expected from the number of pairs of objects with angular distances in the interval  $(\gamma_i - 0.5\delta, \gamma_i + 0.5\delta]$ ,  $i = 1, \dots, N_{\text{bins}}$ , where  $\gamma_i \equiv (i - 0.5)\delta$  and  $\delta \equiv 2\gamma_0/N_{\text{bins}}$  is the bin width. The expected frequency distribution is achieved by a large number of Monte Carlo realizations of isotropically distributed objects in  $\Omega_j^{\gamma_0}$ , containing a similar number of objects as in  $\mathcal{C}^j$  (Teixeira 2003; Bernui & Villela 2006). The 2PACF has the property that its mean, obtained by integrating over all separation angles (Chen & Hakkila 1998), is zero. A positive (negative) value of  $\Upsilon_j$  indicates that objects with these angular separations are correlated (anticorrelated), while zero indicates no correlation.

Define now the scalar function  $\sigma : \Omega_j \rightarrow \mathbb{R}^+$ , for  $j = 1, \dots, N_{\text{caps}}$ , which assigns to the  $j$ -cap, centered at  $(\theta_j, \phi_j)$ , a real positive number  $\sigma_j \equiv \sigma(\theta_j, \phi_j) \in \mathbb{R}^+$ . The most natural way of defining a measure  $\sigma$  is through the variance of the  $\Upsilon_j$  function, we thus define (Bernui et al. 2007)

$$\sigma_j^2 \equiv \frac{1}{N_{\text{bins}}} \sum_{i=1}^{N_{\text{bins}}} \Upsilon_j^2(\gamma_i; \gamma_0). \quad (1)$$

To obtain a quantitative measure of the ACS of the GRBs sky map, we cover the celestial sphere with  $N_{\text{caps}}$  spherical caps, and calculate the set of values  $\{\sigma_j, j = 1, \dots, N_{\text{caps}}\}$  using equation (1). Patching together the set  $\{\sigma_j\}$  in the celestial sphere according to a colored scale (where, for instance,  $\sigma^{\text{min}} \rightarrow$  blue,  $\sigma^{\text{max}} \rightarrow$  red) we obtain a sigma map. Finally, we quantify the ACS of a given sigma map by calculating its angular power spectrum. Because the sigma map assigns a real value to each pixel in the celestial sphere, that is,  $\sigma = \sigma(\theta, \phi)$ , one can expand it in spherical harmonics:  $\sigma(\theta, \phi) = \sum_{l,m} A_{lm} Y_{lm}(\theta, \phi)$ , where the set of values  $\{S_l, l = 0, 1, 2, \dots\}$ , defined by  $S_l \equiv (1/(2l+1)) \sum_{m=-l}^l |A_{lm}|^2$ , is the angular power spectrum of the sigma map. Because we are interested in the large-scale angular correlations, we concentrate on  $\{S_l, l = 1, 2, \dots, 10\}$ . Notice that we are interested in the angular power spectrum of the sigma map, and not that of the celestial sphere where the GRB events are located; this later case was already done by Briggs et al. (1996) and Tegmark et al. (1996). As we show below, the sigma map analysis is able to reveal very small anisotropies, like those induced by the BATSE's sky exposure, despite the small burst detection rate of the SGRBs.

### 4. DATA ANALYSES AND RESULTS

In this section we apply the sigma-map method explained in the previous section to study the large-scale ACS present in the angular distribution of the 516 SGRBs listed in the catalog  $\mathcal{C}$ . In the following, all the sigma maps were calculated using spherical caps of  $\gamma_0 = 90^\circ$  of aperture, that is, hemispheres (smaller spherical caps have less SGRBs in each  $\mathcal{C}^j$ ,  $j = 1, \dots, N_{\text{caps}}$ , hence produce large statistical noise in the  $\Upsilon_j$  functions). We also used  $N_{\text{bins}} = 90$  and  $N_{\text{caps}} = 768$  in these analyses.

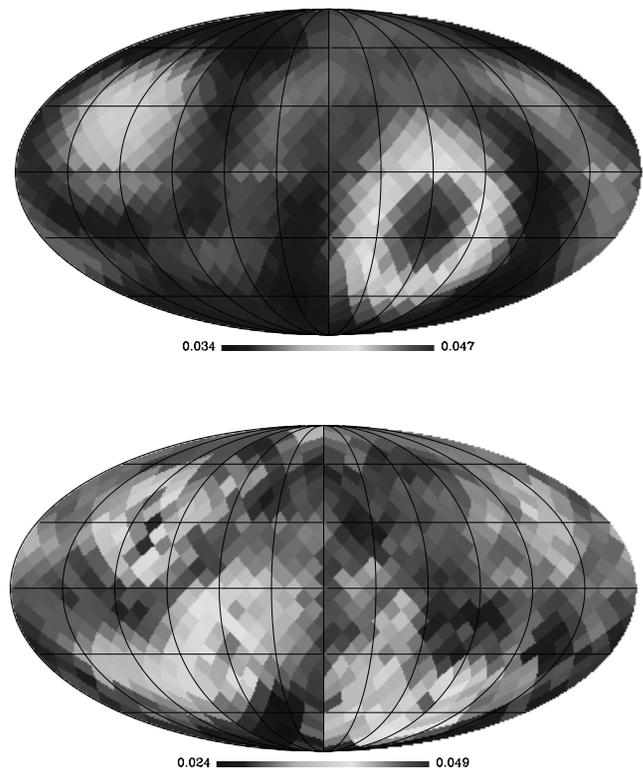


FIG. 1.—Sigma maps in galactic coordinates and with a graticule of  $30^\circ$ . *Top*: This map is the average of 100 of sigma maps each computed from a Monte Carlo simulation of the sky distribution of SGRB events according to the NUSE function. *Bottom*: This is the sigma map calculated from the catalog  $\mathcal{C}$  of BATSE SGRBs data. [See the electronic edition of the Journal for a color version of this figure.]

An important issue that deserves close inspection is the presence of anisotropic ACS in the data induced by the nonuniform sky exposure (NUSE) during the BATSE experiment (Hakkila et al. 1998), expected because some latitudes of the sky were over-observed while others were underobserved. Because there are no reports quantifying or tracing out the influence of the NUSE at large angular scales in the current BATSE catalog 4Bc (see Chen & Hakkila (1998) for analyses of the 3B and 4B catalogs) it is interesting to use the sigma-map method to investigate the possible anisotropic angular correlations that may be present in the data even if their magnitudes are small. For this, our strategy to reveal the large-scale ACS in the data runs in three steps. First, we produce 1000 Monte Carlo maps simulating the sky positions of 516 cosmic objects according to the NUSE function (Hakkila et al. 1998), then we calculate in each case the corresponding sigma map, and finally we compute the angular power spectrum  $\{S_l, l = 1, \dots, 10\}$  of each of these sigma maps. Second, we generate 1000 Monte Carlo maps simulating the sky positions of 516 isotropically distributed cosmic objects, then we compute for each of these Monte Carlo maps their corresponding sigma maps, and finally we calculate the angular power spectrum of these sigma maps. Third, we calculate the sigma maps, and their respective angular power spectrum, of the SGRBs listed in catalog  $\mathcal{C}$ .

In Figure 1 we show two sigma maps in Galactic coordinates. In the top panel we show the average of 100 sigma maps, randomly chosen in between the 1000 sigma maps computed from a similar number of Monte Carlo sky maps which simulate different catalogs of SGRB according to the NUSE function. In the bottom panel we exhibit the sigma map corresponding to the catalog  $\mathcal{C}$ .

In Figure 2 we display a comparative analyses, taking into account isotropic and nonisotropic cases, of the angular power

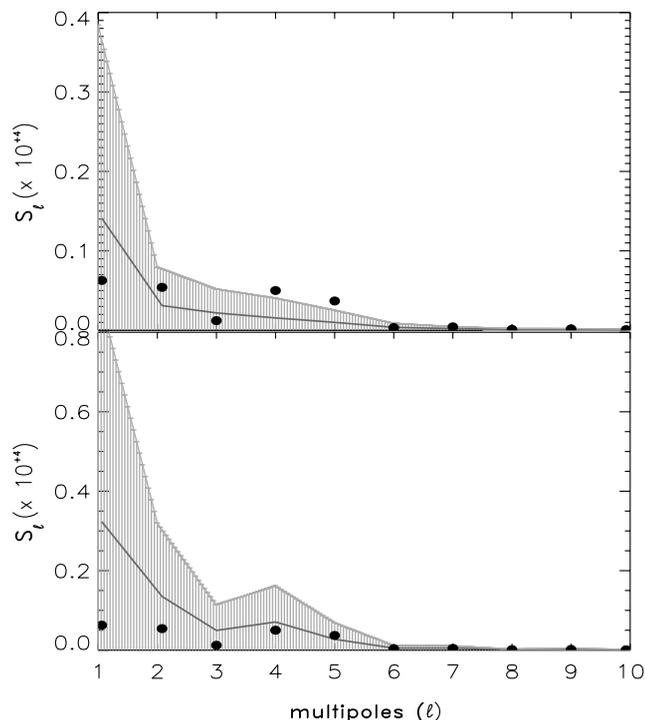


FIG. 2.—Averaged angular power spectra of 1000 sigma maps computed from equal number of Monte Carlo maps, which were produced considering two cases: isotropic (*top*) and anisotropic (*bottom*) hypotheses. For the anisotropic case, the Monte Carlo maps were generated using the NUSE function. Together we plot the angular power spectrum of the sigma map, represented as bullets, corresponding to the BATSE SGRBs listed in the catalog  $\mathcal{C}$ . The shadowed regions correspond to 2 standard deviations in each case. [See the electronic edition of the *Journal* for a color version of this figure.]

spectrum of the sigma map obtained from the angular distribution of the SGRBs listed in  $\mathcal{C}$ . In the top panel, we plotted the angular power spectrum  $S_l$  versus  $l$  of the sigma map obtained from the catalog  $\mathcal{C}$ , together with the mean of 1000 sigma maps computed from an equal number of statistically isotropic Monte Carlo sky maps. In the bottom panel, the plot is similar except that the Monte Carlo sky maps have anisotropic ACSs because were produced considering the NUSE function of BATSE. In both plots the shadowed areas correspond to 2 standard deviations. Besides some small differences, the angular power spectra corresponding to the sigma maps computed from the SGRBs show a very similar large-scale structure when compared with the mean angular power spectrum of the sigma maps obtained from Monte Carols produced according to the NUSE function.

A comparative analysis of the ACS corresponding to these cases, isotropic and nonisotropic due to NUSE function, is better seen if we plot  $l(l+1)S_l$  versus  $l$ , as shown in Figure 3. There we display the corresponding data from the SGRBs together with the mean of the angular power spectra of the isotropic and nonisotropic cases, where now the shadowed area corresponds to 2 standard deviations of the isotropic case. As observed, the data have a very similar behavior to the nonisotropic case, and is different from the isotropic case which shows a flat spectrum. Thus, data and simulated isotropic maps are incompatible at 95% confidence level. However such differences are mainly restricted to the angular scales  $36^\circ$ – $45^\circ$ , which exactly reproduce the imprints exhibited by the angular power spectrum of the nonisotropic case. In the absence of ACSs other than those expected by the NUSE of the BATSE experiment, this result strongly suggests that the SGRBs are intrinsically isotropic.

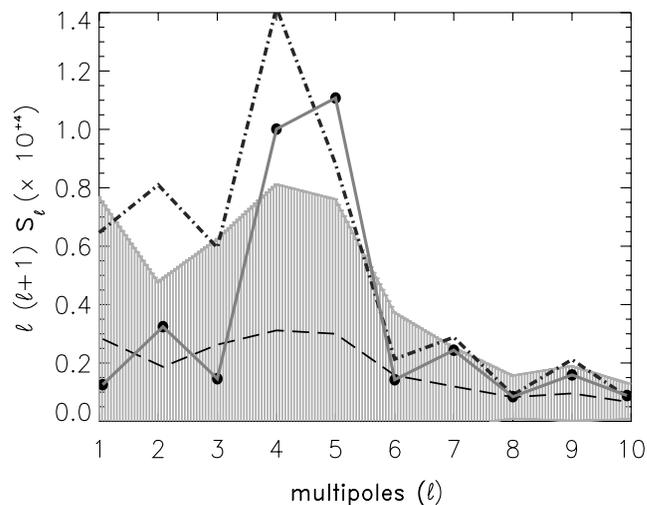


FIG. 3.—Plot of the angular power spectra in the form  $l(l+1)S_l$  vs.  $l$ . The dashed (dot-dashed) line corresponds to the mean angular power spectrum of 1000 sigma maps, each one computed from a Monte Carlo isotropic map (anisotropic map, according to the NUSE function). The shadowed region corresponds to 2 standard deviations of the isotropic case. Together with these data we plotted the angular power spectrum of the SGRBs listed in catalog  $\mathcal{C}$ , represented as bullets. [See the electronic edition of the *Journal* for a color version of this figure.]

To test the robustness of our calculations we also performed the sigma map analyses with  $N_{\text{bins}} = 180$  and  $N_{\text{caps}} = 3072$ , obtaining the same result.

Furthermore, we also searched for a possible correlation between the SGRBs listed in  $\mathcal{C}$  with the supergalactic plane, where nearby galaxies appear to be more concentrated. Because we do not know how many events could be originated in these galaxies, we generate three sets of 300 Monte Carols considering in each case a different number of simulated GRBs provided by an anisotropic distribution which selects events near the supergalactic plane. That is, we generated sets of maps where 33%, 50%, and 66% of the events were produced by such anisotropic distribution, respectively. We then computed their corresponding sigma maps in order to compare them with the sigma map calculated from the SGRBs listed in  $\mathcal{C}$ . To measure such a possible correlation we computed the linear Pearson correlation coefficient between the sigma map of the SGRBs and each one of the sigma maps obtained from these sets of Monte Carlo realizations. Notice that such a coefficient varies from 0 (for totally uncorrelated maps) to 1 (for fully correlated maps). Our results show that the Pearson's coefficient is, in mean, less than 0.03 (using 300 Monte Carols for each of the three above mentioned cases). To realize whether this value is statistically significant, we computed the Pearson's coefficient in some illustrative cases. For example, the mean Pearson's coefficient correlating one sigma map, coming from a given set of sigma maps computed from the above mentioned 66% anisotropic Monte Carlo maps, with the rest of sigma maps from such set is  $0.23 \pm 0.16$  (the result is similar in the other two cases). On the other hand, the mean Pearson's coefficient correlating one sigma map, chosen randomly from the set of 1000 sigma maps calculated from Monte Carlo maps produced according to the NUSE function, with the rest of sigma maps of this set is  $0.25 \pm 0.16$ . Similarly, the mean Pearson's coefficient correlating any sigma map, from the set of 1000 sigma maps computed from Monte Carlo statistically isotropic maps, with the rest of sigma maps of this set is  $0.15 \pm 0.12$ .

Moreover, comparing the sigma map computed from BATSE SGRBs catalog  $\mathcal{C}$  with each of the 1000 sigma maps, obtained from Monte Carlo maps generated according to the NUSE function,

the mean Pearson's coefficient is  $0.14 \pm 0.11$ . A similar analysis of the sigma map of BATSE SGRBs but now considering those Monte Carlo maps generated under the statistical isotropy hypothesis, tells us that the Pearson's coefficient is, in mean,  $0.12 \pm 0.09$ . Taken together this information suggests that there is no evidence for a large-scale angular correlation between BATSE SGRBs and simulated maps produced considering different amounts of events coming from an anisotropic distribution that selects positions in the supergalactic plane and its surrounds. Notice that this result does not contradict the correlation found by Ghirlanda et al. (2006) which is valid for small angular distances  $\leq 3^\circ$ , while the present analysis concerns angular scales  $\geq 18^\circ$ .

Finally, we also tested the robustness of our results under the change of the angular coordinates of the BATSE SGRBs due to the measured error boxes (Briggs et al. 1998). This was done by sorting their angular coordinates within the limits given by the error boxes ( $\pm 2.5^\circ$ ) we found no measurable difference with respect to the results presented here.

## 5. CONCLUSIONS

The purpose of this study is to know the large-scale angular correlations of the set of 516 BATSE SGRBs, and to discover if these correlations are compatible with a statistically isotropic distribution of events, or instead they reveal the ACS resulting as a consequence of the NUSE function of BATSE experiment. To elucidate this, we need to know the angular power spectra of two sets of sigma maps: one set is computed from statistically isotropic Monte Carlo maps and the other is calculated from Monte Carlo maps that simulate the sky position of the events using the NUSE function of BATSE experiment. After that, we compare the power spectra of these sigma maps with the angular power spectrum of the sigma map computed from the BATSE SGRBs data.

The first thing to be noticed in the angular power spectrum of the BATSE SGRBs, plotted in Figure 2, is the absence of dominant anisotropies at the largest angular scales  $60^\circ - 180^\circ$  ( $l = 1,$

2, 3), and a similar situation for angular scales  $\leq 30^\circ$  ( $l \geq 6$ ). However, peculiar features appear at the angular scales  $36^\circ - 45^\circ$  ( $l = 4, 5$ ), and for a better understanding of what information is encoded there we plot these data in the form  $l(l+1)S_l$  versus  $l$  (see Fig. 3). In fact, Figure 3 reveals a second interesting thing, that is, the nonflat spectrum of the BATSE data (represented by bullets) which clearly differs from the flat angular power spectra showed by the statistically isotropic Monte Carlo data (*dashed line*). We also observe in Figure 3 that the mean of the angular power spectra of the sigma maps computed from Monte Carlo maps generated according to the NUSE function (*dot-dashed line*) has also a nonflat spectrum which is very similar to the corresponding one obtained from BATSE SGRBs. In other words, the large-scale angular correlations of the BATSE SGRBs exhibit the anisotropic imprints expected in the data due to the NUSE of BATSE experiment. Other ACS are not found to be statistically significant, at 95% C. L. In conclusion, these results strongly suggests that the SGRBs are intrinsically isotropic.

Finally, we also studied the possible large-angle correlation between the SGRBs data and Monte Carlos with (different amounts of) simulated events concentrated toward the supergalactic plane. No evidence for such large-scale anisotropy was found in the BATSE SGRBs.

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