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Experimental tests on re-entrant klystron cavity for a gravitational wave antenna

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Abstract
The present work quantifies the dependence of the tunable frequency range on the gap spacing between the end of the conical insert and the cavity plate in re-entrant 1.0 GHz klystron cavities. Fabricated from aluminium, the cavities tested are 80 mm in diameter with the top plate 1 mm thick. Experiments performed on such cavities have shown tuning coefficients (change in resonant frequency due to variation of the capacitive gap) as high as 40.0 MHz μm^{-1}, thereby demonstrating the capability of re-entrant cavities as electromechanical transducers in resonant mass gravitational wave antennas. In cavity-based transducers ten times as small as the cavities tested here, like the type used on the gravitational wave antenna Niobé, this result translates into a tuning coefficient 100 times higher.

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1. Introduction

In this paper we examine both theoretically and experimentally the resonance properties of azimuthally symmetric re-entrant cavities. Such devices are used as parametric transducers to continuously monitor the vibrational state of mass gravitational wave antennas through modulation of the capacitive gap [1]. Here we investigate the relationship between the resonant frequency and the cavity dimensions with emphasis on how the frequency varies when flexing the top plate by means of a directed force.

2. Theory and cavity analysis

Featuring a coaxial conical insert, the re-entrant cavity we shall study is pictured in figure 1. On condition that the gap spacing d is much smaller than the resonant wavelength the concept of lumped circuit elements becomes meaningful, whereby the cavity can be treated

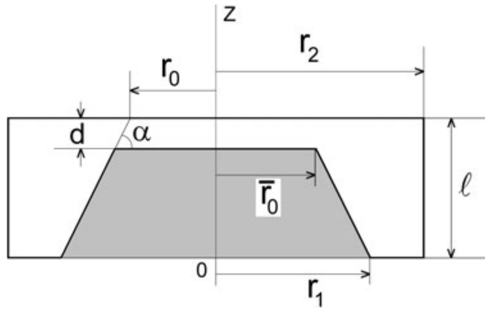

Figure 1. Definition of geometrical parameters for the re-entrant cavity with coaxial conical insert.

as a shorted coaxial line terminated by a capacitor. A lumped circuit analysis of this structure leads to the following equivalent LC parameters [2]:

$$\frac{C_0}{\varepsilon_0} = \frac{\pi \bar{r}_0^2}{d}, \qquad \frac{L}{\mu_0} = \frac{\ell}{2\pi}\left(\ln\frac{e r_2}{r_1} - \frac{r_0}{r_1 - r_0}\ln\frac{r_1}{r_0}\right), \qquad \alpha = \tan^{-1}\frac{\ell - d}{r_1 - \bar{r}_0} \tag{1}$$

$$\frac{C_1}{\varepsilon_0} = \frac{\pi\left(r_0^2 - \bar{r}_0^2\right)}{d} + \frac{2\pi}{\alpha}\left(r_0 \ln\frac{e\ell_M \sin\alpha}{d} + \frac{d\cot\alpha}{2}\ln\frac{\sqrt{e}\ell_M\sin\alpha}{d}\right), \tag{2}$$

where

$$\ell_M = \frac{\sqrt{[2(r_1 - r_0)^2 + 3(r_2 - r_1)(r_1 + r_2 - 2r_0)]^2 + \ell^2(3r_2 - 2r_1 - r_0)^2}}{3(2r_2 - r_1 - r_0)} \tag{3}$$

with the resonant frequency $f_0 = 1/2\pi\sqrt{L(C_0 + C_1)}$. The capacitance C_0 accounts for the gap region, where the electric field lines run axially from the top surface of the post to the upper plate. Far from the gap and close to the bottom of the cavity, the electric field lines spread out from the lateral surface of the post radially with the corresponding annular volume being represented by the inductance L, which expresses the magnetic flux that links the inner coaxial post and the external cylinder. The transitional region between the side wall and the upper gap is embodied by the capacitance C_1 describing the fringing fields that develop in the vicinity of the upper edges of the cavity. Accordingly, the accuracy of the formulae becomes better for larger r_0/ℓ_M and smaller $\ell_\mathrm{M}/\lambda_0$, respectively the radius of the post and the resonant wavelength compared to the relative size of the cavity [2].

3. Experiment

The resonance properties of a re-entrant cavity with conical insert are experimentally investigated by looking at the effect on the resonant frequency of reducing the gap spacing through application of a bending force at the centre of the circular top plate with clamped edges. Resonant frequencies are measured by the reflection measurement method described in [3], in which the cavity fields are both excited and detected by means of a single electric probe inserted through a 1.0 mm diameter hole drilled halfway across the cylindrical wall. In the calculation of the resonant frequencies, the dynamical gap used in (1)–(3) is taken as the nominal gap d (figure 1) minus the deflection $\delta_{\max}$ produced by an external force acting on the top plate (figure 2). The deformation of the plate is determined from the following expression that gives the deflection due to pure bending of a clamped circular plate loaded at the centre [4]:

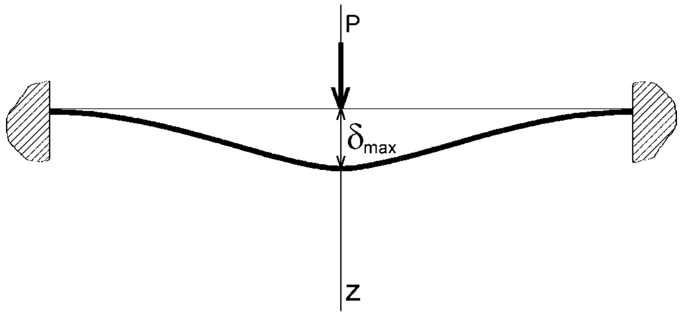

Figure 2. Deflection of a clamped plate loaded at the centre.

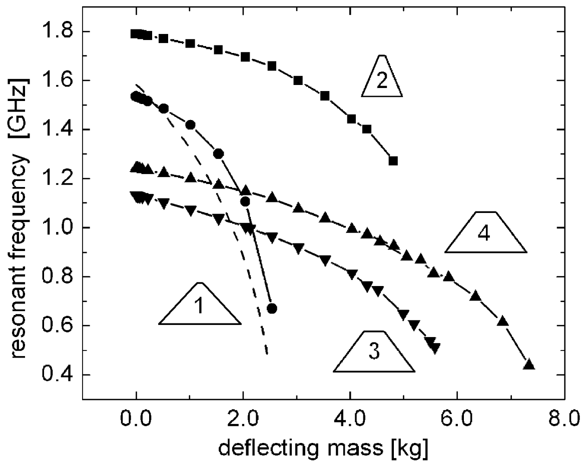

Figure 3. Measured resonant frequencies for four conical inserts. The dashed curve refers to calculated frequencies for insert 1.

$$\delta(r, P) = \frac{Pr^2}{8\pi D} \ln \frac{r}{r_2} + \frac{P}{16\pi D}\left(r_2^2 - r^2\right), \qquad (4)$$

where P is the applied load, $D = Eh^3/12(1-\nu^2)$ denotes the flexural rigidity of the aluminium plate of thickness $h = 1.0$ mm, modulus of elasticity $E = 69.0$ GPa and Poisson's ratio $\nu = 0.3$.

Four coaxial inserts (with dimensions given in table 1) have been tested on a hollow circular cavity of radius $r_2 = 40.0$ mm and height $\ell = 20.0$ mm. On applying a deflection force (using a set of calibrated weights) we then measured the corresponding downshifted frequencies, which are plotted in figure 3. Comparing inserts 3 and 4, we see that the corresponding tuning curves are somewhat similar, differing by a frequency shift controlled by the gap spacing d. Insert 1 having the same r_1 as inserts 3 and 4 but with smaller $\bar{r}_0$ and d produces the steepest curve, with its lowest part yielding tuning coefficients as high as 40.0 MHz μm^{-1}. Insert 2, with smaller r_1 and larger d, has the effect of flattening the curve relative to curve 1. Regarding the dashed line in figure 3, this gives calculated frequencies for insert 1 by using expressions (1)–(4). We see that the calculated curve well fits frequencies measured at lower deflecting masses and deviates apart when the bending force increases. This is reasonable as calculation assumes plane parallel surfaces separated $d - \delta_{max}$ apart,

Table 1. Gap spacing (d), bottom (r_1) and top ($\bar{r}_0$) radii of the inserts tested.

Insert	d (mm)	r_1 (mm)	$\bar{r}_0$ (mm)
1	0.10	20.0	1.5
2	0.15	10.0	1.5
3	0.30	20.0	5.0
4	0.40	20.0	5.0

while in the actual experiment the plate takes on the shape of a concave surface as pictured in figure 2.

4. Conclusions

We have discussed the feasibility of a 1.0 GHz re-entrant cavity as a parametric transducer by demonstrating experimentally the transducer sensitivity to deflections of the 80.0 mm diameter, 1.0 mm thick aluminium plate when loaded with weights as light as 10 g. Through proper selection of the cavity geometry by increasing r_1 (with r_2 and λ fixed) and reducing both $\bar{r}_0$ and the gap d, a tuning coefficient of $\Delta f \Delta d = 40.0$ MHz μm^{-1} has been achieved at a gap spacing of 0.1 mm. In addition to conferring a high tuning coefficient, a narrow gap maximizes the electrical coupling to a mechanical transformer that matches the impedance of the transducer to the antenna impedance. Scaled up by a factor of 10 for ease of machining and testing in the early stages of the design, the cavities we have examined are developmental replicas of electromechanical transducers to be instrumented in a resonant mass gravitational wave antenna under development at INPE [5]. In this experiment the cavity-based transducer, which actually operates at 10.0 GHz, is expected to be a hundred times more sensitive than the oversized cavities tested here. To put into perspective a displacement sensitivity of 4000 MHz μm^{-1}, we mention that a similar 9.6 GHz re-entrant cavity with the sensitivity of 345 MHz μm^{-1} has been implemented as a practical transducer on the resonant bar antenna Niobè [6].

Acknowledgments

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